**3 Explanatory notes on guideline for direct stability assessment**

**3.1 Background of guideline for direct stability assessment procedure**

**3.1.1 Nomenclature**

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| Bwl, m | waterline breadth of ship |
| d, m | mean draught of ship |
| Fr=vs(gLpp)0.5 | Froude number |
| GM, m | metacentric height of ship |
| hr, m | height of considered location above assumed roll axis |
| hs, m | significant wave height |
| kxx, m | dry roll radius of inertia with respect to centre of gravity |
| kyy, m | dry pitch radius of inertia with respect to centre of gravity |
| kzz, m | dry yaw radius of inertia with respect to centre of gravity |
| Lpp, m | length of ship between perpendiculars |
| N | number of simulations |
| fs, (m⋅s)‑1 | joint probability density of sea state, i.e. probability of sea states per unit range of significant wave heights and mean zero-upcrossing wave periods |
| r, 1/s | mean rate of stability failures (mean number of stability failures per time) |
| T, s | mean time until stability failure |
| Tr, s | linear natural roll period of ship in calm water |
| Tz, s | mean zero-upcrossing wave period |
| vs, m/s | ship forward speed |
| , degree | roll angle (positive for starboard down) |
| 3h | mean 3 hour maximum roll amplitude |
| μ, degree | wave direction (0 degree for following waves, 90 for waves from steering board and 180 for head waves) |
| r, rad/s | linear natural roll frequency of ship |

**3.1.2 Definition of stability failure**

3.1.2.1 Exceedance of a threshold roll angle and a threshold lateral acceleration are used as stability failures; namely, unless stricter requirements are deemed to be necessary for particular ships or ship types, the following definitions seem appropriate:

.1 *exceedance of roll angle* defined as the minimum of 40 degrees, angle of vanishing stability in calm water and angle of submergence of unprotected openings in calm water; or

.2 *exceedance of lateral acceleration* of 9.81 m/s2.

3.1.2.2 To simplify the evaluation of motion criteria, instead of the requirement in paragraph 3.1.2.1.2, an equivalent maximum acceptable roll angle, defined as 57.3/(1+hrr2/9.81), in degree, can be used. For this calculation, the roll axis can be assumed at the midpoint between the waterline and the centre of gravity of the ship.

3.1.2.3 Thus, in numerical simulations, only one stability failure event will need to be tracked: exceedance of the minimum of the three roll angles defined in 3.1.2.1.1 and 3.1.2.2.

**3.1.3 Introduction**

3.1.3.1 In a probabilistic direct stability assessment, probability of stability failure is used directly as a safety measure (criterion), therefore, such assessment requires some form of counting of stability failures, which hence need to be encountered in the simulations. This leads to the problem of rarity, because very long simulations are required for the relevant ships and loading conditions. Besides, reliable estimation of the mean stability failure probability requires simulation of a sufficiently large number of stability failures, which further increases the required simulation time.

3.1.3.2 At the same time, direct stability assessment should enable most accurate assessment within SGISC, taking into account as much relevant physics as possible in the most accurate way. This means that the simulation tools employed are slow and require much more computational time than tools used in level 1 and level 2 vulnerability assessment. Therefore, some simplifications are required regarding probabilistic procedures. Here, three such simplification methods are exploited.

**3.1.4 Ships and loading conditions used in tests**

3.1.4.1 Five ships were used: a cruise and a RoPax vessels and three container ships of 1700, 8400 and 14000 TEU capacity. For each ship, 5 loading conditions were selected: three loading conditions with small GM values, relevant for parametric roll, pure loss of stability and stability in dead ship condition, and two loading conditions with big GM values, relevant for excessive accelerations, Table 3.1.1. To fine-tune the ranges of the tested GM values, level 1 and level 2 vulnerability assessments regarding all stability failure modes were conducted.

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| Table 3.1.1 Ships and loading conditions used in study | | | | | | | | |
| Ship | Lpp, m | Bwl,m | Loading condition: | 01 | 02 | 03 | 04 | 05 |
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| Cruise vessel | 230.9 | 32.2 | d, m | 6.9 | | | | |
| GM, m | 1.5 | 2.0 | 2.5 | 3.25 | 3.75 |
| RoPax vessel | 175.0 | 29.5 | d, m | 5.5 | | | | |
| GM, m | 3.7 | 4.5 | 5.2 | 5.9 | 6.6 |
| 1700 TEU container ship | 159.6 | 28.1 | d, m | 9.5 | | | 5.5 | |
| GM, m | 0.5 | 1.2 | 1.9 | 5.75 | 6.75 |
| 8400 TEU container ship | 317.2 | 43.2 | d, m | 13.93 | 14.44 | 14.48 | 11.36 | |
| GM, m | 0.89 | 1.26 | 2.01 | 5.0 | 6.93 |
| 14000 TEU container ship | 349.5 | 51.2 | d, m | 14.5 | | | 8.5 | |
| GM, m | 1.0 | 2.0 | 3.0 | 9.0 | 12.0 |

**3.1.5 Database of results of direct simulations**

3.1.5.1 For each ship and each loading condition, full probabilistic assessment was performed using numerical simulations of ship motions in waves to provide validation database for simplified procedures. The simulations were performed for six forward speeds, Table 3.1.2.2, for the mean zero-upcrossing wave periods Tz and significant wave heights hs covering all entries in the North Atlantic wave scatter table, IACS Rec. 34, and for wave directions μ from 0 to 180 degrees every 10 degrees.

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| Table 3.1.2.2 Non-dimensional forward speeds used in analysis | | | | | | | |
| Ship | Lpp, m | Froude numbers | | | | | |
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| Cruise | 230.9 | 0.0 | 0.0454 | 0.0908 | 0.1362 | 0.1816 | 0.2270 |
| RoPax | 175.0 | 0.0 | 0.0546 | 0.1093 | 0.1639 | 0.2185 | 0.2732 |
| CV1700 | 159.6 | 0.0 | 0.0481 | 0.0962 | 0.1443 | 0.1924 | 0.2405 |
| CV8400 | 317.2 | 0.0 | 0.0452 | 0.0904 | 0.1356 | 0.1808 | 0.2259 |
| CV14000 | 349.5 | 0.0 | 0.0427 | 0.0854 | 0.1281 | 0.1708 | 0.2135 |

3.1.5.2 For each combination of forward speed, wave period, significant wave height and wave direction, numerical simulations of ship motions in 200 realisations of the same sea state were performed by random variation of frequencies, directions and phases of wave components composing sea state. Each simulation was conducted for the simulation time 1.7⋅104 hours or until the first exceedance event, after which it was repeated in another realisation of the same seaway.

3.1.5.3 From each simulation, the time until stability failure Ti was defined; the estimate of the mean time until stability failure T was calculated by averaging over N=200 failures as

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|  |  | (3.1.1) |

3.1.5.4 The maximum likelihood estimate for the rate r, 1/s, of stability failures is

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|  |  | (3.1.2) |

3.1.5.5 Note other useful relationships:

.1 probability that at least one failure happens during time t is

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|  |  | (3.1.3) |

.2 standard deviation of time until stability failure is

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|  |  | (3.1.4) |

.3 variance of time until stability failure is

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|  |  | (3.1.5) |

3.1.5.6 The studied ships demonstrated stability failures due to principal parametric resonance in bow waves, principal and fundamental parametric resonance in stern waves and synchronous roll in beam waves (relevant for dead ship and excessive acceleration stability failures). Some of loading conditions indicated big heel angles in following waves at large forward speeds, although their maximum speeds, while sufficient for vulnerability to the pure loss of stability, were not high enough for strong pure loss failures. Surf-riding/broaching was not found relevant for any of the tested ships.

3.1.5.7 To test and validate simplified probabilistic procedures, including extrapolation of the stability failure rate over wave height, design situations and non-probabilistic assessment, it was necessary to separate the stability failure events identified in the direct simulations with respect to stability failure modes. Although the extrapolation of stability failure rate over wave height does not assume any specific stability failure mechanism and is applicable to any stability failure mode, it was interesting to check how much its accuracy and robustness differ between different stability failure modes. On the other hand, in the document SDC 3/INF.12 it was found that the same design situations cannot be used for different stability failure modes, therefore, different failure modes require different design situations, which requires the definition of the failure mode-specific stability failure rate in the full probabilistic assessment for validation and calibration of the failure mode-specific desing situations.

3.1.5.8 In the full probabilistic assessment, parametric roll (specifically, principal parametric resonance) in bow waves was detected in mean wave directions from head up to about 70 degree off-bow; nevertheless, in all cases where principal parametric resonance in bow waves occurred, head waves led to largest roll motions, Figure 3.1.1 (top left and top middle plots). Therefore, for parametric roll in bow waves, assessment in head waves will always detect the worst situations and, moreover, include most relevant stability failure events. Therefore, to select the relevant simulation results from the full database for validation and calibration of simplified methods for parametric resonance in bow waves, three sets of reference data were generated, for wave directions from 170 to 180, 160 to 180 and 150 to 180 degree.

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| Figure 3.1.1. Colour plots of mean three-hour maximum roll amplitude depending on mean wave period (in s, radial coordinate) and mean wave direction (circumferential coordinate, waves from top, bottom and right correspond to 180, 0 and 90 degree, respectively) for principal parametric resonance at low (left) and medium (middle) GM and synchronous roll at high GM (right) at low (top) and high (bottom) forward speed | | |

3.1.5.9 Parametric resonance (principal and, much less, fundamental) in stern waves was detected in the full probabilistic assessment in wave directions from following up to about 80 degree off-stern. Unlike for parametric roll in bow waves, for which head waves always represent the worst case, following waves were not always worst (over all stern wave directions) for parametric roll in stern waves. Moreover, for some loading conditions at certain forward speeds, parametric roll did not occur in following waves while being very strong in stern-quartering waves, Figure 3.1.1 (bottom left and middle); see a detailed discussion in Shigunov (2009)[[1]](#footnote-1). This means that for some ships in some loading conditions, assessment in following waves may not detect the possibility of severe parametric roll in stern waves.

3.1.5.10 This is unpleasant since the need to address parametric roll in stern-quartering wave directions in simplified assessment procedures can lead to the following problems:

.1 since level 1 and level 2 vulnerability assessment do not consider parametric resonance in stern-quartering waves, direct stability assessment including stern-quartering wave directions may lead to inconsistency;

.2 number of required design situations will significantly increase if assessment of parametric roll in stern waves will require all wave directions from following to 90 degree off-stern; moreover, this means significantly more expensive model tests and much more advanced model testing facilities required.

3.1.5.11 To check whether addressing parametric roll specifically in stern-quartering waves is essential for direct assessment, the results of the full assessment are plotted in Figure 2 in the following way: y-axis corresponds to the total stability failure rate over all wave directions, whereas x-axis corresponds to the sum of stability failure rates over parametric roll in bow and stern waves (sectors from 150 to 180 and 0 to 30 degree, respectively) and synchronous roll in beam waves (60 to 120 degree) for all ships and loading conditions (differentiated by symbol type and colour) and forward speeds; thus, x-axis variable neglects parametric roll in stern-quartering waves, included in the y-axis variable.

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|  | Figure 3.1.2. Total stability failure rate in all wave directions vs. sum of stability failure rates due to parametric roll in bow and stern waves and synchronous roll in beam waves; symbol type and colour differentiate ships and loading conditions |

3.1.5.12 Since the dependency in Figure 3.1.2 is monotonous and rather sharp, contributions from parametric resonance in stern-quartering waves do not need to be additionally addressed in the direct stability assessment (unlike in operational measures): taking into account parametric resonance in following waves is sufficient to represent the contributions of parametric resonance in all stern wave directions. The reason is that parametric resonance in stern-quartering waves becomes important with increasing forward speed, when parametric roll decreases, whereas much larger contributions occur in following waves at low forward speeds.

3.1.5.13 Therefore, for validation and testing of the simplified procedures for parametric resonance in stern wave directions, three comparative sets of data were generated from the full database of assessment results, corresponding to wave directions from 0 to 10, 0 to 20 and 0 to 30 degree.

3.1.5.14 For synchronous roll in beam waves, the relevant wave directions in the full probabilistic assessment were found from about 40 degree off-bow to about 40 degree off-stern, depending on the forward speed, Figure 1 (top right and bottom right). However, at low forward speeds, wave directions close to beam are sufficient to assess synchronous roll. Therefore, to select relevant cases for validation for synchronous roll in beam waves from the full database of assessment results, three comparative sets of reference data were generated: for wave directions from 80 to 100, 70 to 110 and 60 to 120 degree.

3.1.5.15 Reference data for pure loss of stability were also generated, although this stability failure was especially difficult to identify, since none of the selected ships was expected to undergo severe pure loss, due to low (although in the region of vulnerability) maximum speeds. Three simple conditions were used: following waves, encounter period (corresponding to peak wave period) exceeding 30 s and wave length, corresponding to the peak wave period, close to the ship length.

**3.1.6 Extrapolation of failure rate over wave height**

3.1.6.1 In SDC 4/5/8 and SDC 4/INF.8, extrapolation of stability failure rate over significant wave height in the form suggested by Tonguc & Söding (1986)[[2]](#footnote-2) was validated for synchronous roll in beam waves (relevant for dead ship and excessive acceleration failure modes),

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|  |  | (3.1.6) |

3.1.6.2 In eq. (3.1.6), T means the expected time to stability failure, hs the significant wave height and A and B constants, independent from the significant wave height but depending on the ship, loading condition, forward speed and wave period and direction.

3.1.6.3 Here eq. (3.1.6) is applied also to parametric roll in bow and stern waves. To quantify the accuracy of the extrapolation, several variants of extrapolation were tested by varying the number of extrapolation points. Namely, 4, 5, …, 11 wave heights were selected, starting from the minimum wave height for which the results could be obtained by direct simulations and for which , i.e. , see document SDC 4/INF.8. All of these points excepting one (corresponding to the minimum significant wave height) were used to perform extrapolation (3.1.6) using 3, 4, …, 10 points, respectively, whereas the results of the direct simulation at the minimum significant wave height was used to find the deviation between the extrapolated and directly computed mean time to failure.

3.1.6.4 Figure 3.1.3 shows the results as histograms of the ratio of the extrapolated to directly computed estimate of the mean time to failure: y-axis corresponds to the number of cases in bins (normed on 1) and x-axis shows the ratio of the extrapolated expected time to failure Textr to the directly estimated one T.

3.1.6.5 To quantify the accuracy of extrapolation, the percentage of the extrapolated values was calculated, lying within the 95%-confidence interval of the directly computed estimate, Table 3 (if the extrapolation were exact, 95% of extrapolated values would have been within this interval). The results show that the extrapolation given by eq. (3.1.6) provides sufficiently accurate results and thus is a useful practical tool to accelerate direct assessment.

**3.1.7 Design situations**

3.1.7.1 The full probabilistic assessment requires summation of short-term stability failure rates over all sea states of a relevant wave climate and all seaway directions and thus large computational time. The document SDC 3/INF.12 proposed to reduce the assessment to few combinations of sea state parameters (wave height, period and direction) and ship forward speed, referred to as design situations.

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| Figure 3.1.3. Histogram (number of cases normed on 1) of ratio Textr/T and 95%-confidence interval of directly computed T (vertical lines) for (from top to bottom) parametric roll in bow waves, parametric roll in stern waves, synchronous roll in beam waves, pure loss of stability (bottom left and middle) and all cases together (bottom right); different symbols correspond to various number of points used in extrapolation over wave height |

3.1.7.2 The idea is that a simplified safety criterion can be used for norming if the dependency of the true long-term probability of stability failure on this criterion (a) is monotonous and (b) shows little scatter between different ships, loading conditions and forward speeds. The standard for this simplified criterion (further referred to as threshold to differentiate it from the long-term standard) can be defined using a sufficient number of representative case studies, Figure 4. Thus the exact dependency w(s) does not matter in the practical approval and is not required, as long as it is proven that it satisfies conditions (a) and (b).

3.1.7.3 Document SDC 3/INF.12 proposed to use different design situations for different failure modes; in SDC 4/5/8 and SDC 4/INF.8, this method was verified for roll in beam sea (to address dead ship condition and excessive acceleration stability failure modes). Here, the verification is extended to other stability failure modes.

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| Table 3.1.3. Percentage of extrapolated values of time to stability failure within 95%-confidence interval of directly computed estimate | | | | | | | | |
| Number of wave heights used for extrapolation | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
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| **Parametric resonance in bow waves** | | | | | | | | |
| Wave directions 150 to 180 degree | 79 | 83 | 85 | 84 | 83 | 81 | 78 | 81 |
| Wave directions 160 to 180 degree | 79 | 82 | 84 | 82 | 81 | 79 | 77 | 79 |
| Wave directions 170 to 180 degree | 78 | 82 | 83 | 81 | 80 | 78 | 77 | 76 |
| **Parametric resonance in stern waves** | | | | | | | | |
| Wave directions 0 to 10 degree | 79 | 82 | 80 | 76 | 73 | 75 | 71 | 62 |
| Wave directions 0 to 20 degree | 79 | 83 | 84 | 81 | 78 | 80 | 79 | 68 |
| Wave directions 0 to 30 degree | 79 | 82 | 81 | 79 | 76 | 78 | 76 | 68 |
| **Synchronous resonance in beam waves** | | | | | | | | |
| Wave directions 70 to 110 degree | 77 | 83 | 85 | 87 | 88 | 88 | 85 | 77 |
| Wave directions 50 to 130 degree | 77 | 82 | 83 | 85 | 85 | 85 | 82 | 74 |
| Wave directions 30 to 150 degree | 77 | 82 | 83 | 84 | 84 | 84 | 82 | 78 |
| **Pure loss in following waves** | 77 | 82 | 83 | 84 | 84 | 86 | 87 | 88 |
|  | | | | | | | | |
| **All above cases** | **77** | **81** | **82** | **83** | **82** | **81** | **79** | **75** |

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|  | Figure 4. Idea of simplified safety criterion s; w is the “true” safety measure, e.g. mean long-term probability of stability failure |

3.1.7.4 To verify conditions (a) and (b) in 7.1, the mean long-term rate of stability failures was computed using the results of the full probabilistic assessment as

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|  |  | (7) |

3.1.7.5 In eq. (7), vs is the ship forward speed, μ is the wave direction and s=(hs,T1) denotes all sea states in the scatter table. Different forward speeds were applied and evaluated separately, because the selection of a suitable speed to be used in design situations was one of the tasks of this investigation.

3.1.7.6 As the first step, wave directions for design situations were selected: 180 degree for parametric roll in bow waves, 0 degree for parametric roll in stern waves, 90 degree for synchronous roll in beam waves and 0 degree for pure loss of stability.

3.1.7.7 The second step was the selection of wave height (aiming at using only one significant wave height per wave period). Several approaches to the selection of sea states in design situations were compared in SDC 4/5/8 and SDC 4/INF.8, including sea states according to the steepness table from MSC.1/Circ.1200, sea states along constant steepness lines , along lines of constant density of sea state occurrence probability and along lines of constant normed and not normed quantiles of sea state occurrence probability.

3.1.7.8 Results shown in SDC 4/5/8 and SDC 4/INF.8, confirmed here, indicate that sea states selected along the lines of constant density of sea state occurrence probability, Figure 5, provide the best correlation between w and s; therefore, results are shown here only for such design sea states. Note that using design sea states along the lines of constant normed and not normed quantiles of sea state occurrence probability (the latter mean lines of constant conditional exceedance probability of various significant wave heights) results in comparable quality of results. Also note that the lines of constant probability density or constant quantiles of probability were defined using logarithmic interpolation for probabilities.

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|  | Figure 3.1.5 Lines of constant density of sea state occurrence probability fs, (m⋅s)‑1, for North-Atlantic wave scatter table |

3.1.7.9 As the simplified criterion in these sea states, maximum (over all design sea states) stability failure rate r was used, following recommendations in SDC 4/5/8 and SDC 4/INF.8.

3.1.7.10 Figures 3.1.6 to 3.1.9 show the mean long-term stability failure rate w vs. maximum (over design sea states) mean short-term failure rate for design sea states with probability densities of 10-7, 10-6, …, 10-2 (m⋅s)‑1 for all failure modes. Each point corresponds to one ship, loading condition and forward speed.

3.1.7.11 The sharp monotonous dependencies in Figure 3.1.2, concerning selection of wave directions for design situations, and in Figure 3.1.6, Figure 3.1.7, Figure 3.1.8 and Figure 3.1.9 (at fs of 10‑4 (m⋅s)‑1 and less), concerning selection of wave heights for design situations for parametric roll in bow and stern waves, synchronous roll in beam waves and pure loss of stability, respectively, indicate that the accuracy of the simplified criterion is satisfactory and improves with increasing wave steepness. Note that the required model testing or numerical simulation time quickly reduces with the increasing wave height, therefore, it is better to use design sea states of larger steepness; however, sea states of too large steepnesses may be difficult to realise in model tests or numerical simulations.

3.1.7.12 To check whether parametric roll in stern waves can be related to assessment results in design sea states in head waves, which would allow skipping assessment for parametric roll in stern waves, Figure 3.1.10 shows the mean long-term stability failure rate due to parametric roll in stern waves vs. the maximum mean short-term stability failure rate in design sea states in head waves; however, the correlation is very poor.

3.1.7.13 Results presented so far allow reducing the number of assessment cases due to using one wave direction per failure mode (reduction factor of about 19) and one wave height per wave period (reduction factor of several orders of magnitude, because assessment at low wave heights requires very long simulations, if feasible at all). Another reduction possibility is the selection of a suitable forward speed: if, for example, only one speed needs to be used per failure mode, this will lead to a reduction of the number of test cases by about one order of magnitude for some stability failure modes, as well as will allow significant simplifications in numerical simulations or model test setup.

3.1.7.14 For dead ship condition and excessive accelerations, only zero forward speed is applied in the full assessment anyway; for pure loss of stability, the rate of stability failures increases monotonously with increasing speed (for the considered ships), therefore, the maximum possible speed should be used. To select the forward speed for design situations for parametric roll, Figure 3.1.11 (left) shows failure rate for parametric roll in head waves along the fs=10-5 (m⋅s)‑1 line (maximum over all wave periods) as a function of Froude number. Each plot corresponds to one ship, and each line corresponds to one loading condition.

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| Figure 3.1.6 Dependency w(s) for design situations for parametric roll in bow waves: mean long-term stability failure rate w(ship,LC,vs), 1/s, y axis, in wave directions from 170 to 180 degree (left), 160 to 180 degree (middle) and 150 to 180 degree (right) vs. simplified criterion, 1/s, x axis – short-term mean stability failure rate in head waves, maximum over design sea states along lines with sea state probability density fs of (top to bottom) 10-7, 10-6, 10-5, 10-4, 10-3 and 10-2 (m⋅s)‑1 |

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| Figure 3.1.7 Dependencies w(s) for design situations for parametric roll in stern waves: mean long-term stability failure rate w(ship,LC,vs), 1/s, y axis, in wave directions 0 to 10 degree (left), 0 to 20 degree (middle) and 0 to 30 degree (right) vs. simplified criterion, 1/s, x axis – short-term mean stability failure rate in following waves, maximum over design sea states along lines with sea state probability density fs of (top to bottom) 10-7, 10-6, 10-5, 10-4, 10-3 and 10-2 (m⋅s)‑1 |

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| Figure 3.1.8 Dependency w(s) for synchronous roll in beam waves: mean long-term stability failure rate w(ship,LC,vs), 1/s, y axis, in wave directions from 80 to 100 degree (left), 70 to 110 degree (middle) and 60 to 120 degree (right) vs. simplified criterion, 1/s, x axis – short-term mean stability failure rate at μ=90 degree, maximum over design sea states along lines with sea state probability density fs equal to (from top to bottom) 10-7, 10-6, 10-5, 10-4, 10-3 and 10-2 (m⋅s)‑1 |

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| Figure 3.1.9 Dependency w(s) for pure loss of stability: mean long-term stability failure rate w(ship,LC,vs), 1/s, y axis, vs. simplified criterion, 1/s, x axis – short-term mean stability failure rate in following waves, maximum over design sea states with occurrence probability density fs of (left to right, then top to bottom) 10-7, 10-6, 10-5, 10-4, 10-3 and 10-2 (m⋅s)‑1 |

3.1.7.15 The results show that for all loading conditions with high failure rate, the failure rate decreases with increasing forward speed. This is due to, first, broadening of the encounter wave spectrum with increasing forward speed in bow waves and, second, due to increasing roll damping with increasing forward speed. Note also that according to operational experience, parametric roll accidents in bow waves always happen at low forward speed. For RoPax vessel in all loading conditions and cruise vessel in two loading conditions with the largest GM, the stability failure rate increases with increasing forward speed; however, the stability failure rate for these cases is very small anyway. Therefore, it seems appropriate to use only zero forward speed in design situations for parametric roll in bow waves. Note that if zero speed is difficult to implement in model tests (e.g. due to wave reflections) or in simulations, as low as practicable forward speed can be applied.

3.1.7.16 Concerning parametric roll in stern waves, Figure 3.1.11 (right) shows a more complex dependency of the failure rate on the Froude number in design sea states in following waves. This is due to the more complex relationship between the wave frequency and the encounter frequency in stern waves and thus more complex behaviour of the encounter wave spectrum. It appears, however, that in all cases with big stability failure rate, simplified assessment only at zero forward speed will either not introduce any non-conservative error or will be conservative, thus zero (or as low as practicable) forward speed appears appropriate also for parametric roll in following waves.

3.1.7.17 Note that zero forward speed in high head or following waves is impossible in reality because of the inability of a ship (with a usual steering system) to keep course at zero speed; here, however, this assumption is acceptable as a practical simplification of the roll motion assessment procedure (which, however, will require some adjustment of the setup).

3.1.7.18 Reducing assessment of parametric roll to zero forward speed case has also the following effect: Figure 3.1.12 (left) shows stability failure rate due to parametric resonance at zero forward speed in design situations in following (y axis) vs. head (x axis) wave directions: obviously, in the relevant region, these two stability failure rates are well correlated. Note that the full probabilistic assessment with respect to parametric resonance shows the same at zero forward speed, Figure 3.1.12 (right), unlike when all forward speeds were taken into account in Figure 3.1.10. Therefore, assessment with respect to parametric resonance in following waves at zero forward speed can be omitted in the design situations approach.

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| Figure 3.1.10 Dependency w(s) for design situations for parametric roll in stern waves: mean long-term stability failure rate w(ship,LC,vs), 1/s, y axis, in wave directions from 0 to 10 degree (left), 0 to 20 degree (middle) and 0 to 30 degree (right) vs. simplified criterion, 1/s, x-axis – short-term mean stability failure rate in head waves, maximum over design sea states along lines with sea state probability density fs of (top to bottom) 10-7, 10-6, 10-5, 10-4, 10-3 and 10-2 (m⋅s)‑1; unlike in Figure 3.1.7, where simplified criterion is calculated in following waves, here simplified criterion is calculated in head waves |

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| Figure 3.1.11 Maximum (over all wave periods) mean short-term stability failure rate, 1/s, at wave height corresponding to sea state probability density fs=10-5 (in (m⋅s)‑1, y axis) vs. Froude number (x axis) in head (left) and following (right) waves for (from top to bottom) 1700 TEU container ship, RoPax, cruise vessel and 8400 and 14000 TEU container ships; each line corresponds to one loading condition | |

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| Figure 3.1.12. Stability failure rate due to parametric resonance in design situations at zero forward speed (left, symbols differentiate sea state probability density) and in full probabilistic assessment at zero forward speed (right) in following (y axis) vs. head (x axis) waves for all ships and loading conditions |

3.1.7.19 One more possibility to reduce the number of design situations is to specify the wave period (or at least limit the relevant range of wave periods) before performing seakeeping tests or simulations, e.g. based on the natural roll period from a linear estimation or from roll decay simulations or roll decay model tests (which are performed before seakeeping tests anyway).

3.1.7.20 The difficulty is that the natural roll period strongly depends on the roll amplitude: Figure 3.1.13 shows the natural roll period estimated from roll decay simulations as a function of the roll amplitude. The dependencies indicate a non-monotonous behaviour, e.g. a decrease of the natural roll period with increasing roll amplitude at small to moderate roll amplitudes, due to nonlinearity of GZ curve, followed by an infinite growth of the natural roll period when roll amplitude approaches the angle of vanishing stability. The other difficulty is that in irregular waves, there is no perfect characteristic of the excitation frequency.

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|  |  | Figure 3.1.13 Natural roll period vs. roll amplitude from roll decay for five test ships; lines differentiate loading conditions |

3.1.7.21 From the results of numerical simulations for those conditions of design situations that are already defined above (namely, wave direction 180 degree for parametric roll in bow waves, 0 degree for parametric roll in stern waves and 90 degree for synchronous roll in beam waves and zero forward speed in all cases), the zero-upcrossing wave period leading to the maximum failure rate over all design sea states was identified, Table 3.1.4. According to these results, the range of the encounter wave periods (calculated using the mean zero-upcrossing wave period) leading to maximum failure rate can be localised between 0.3 and 0.5 of the linear natural roll period for principal parametric resonance in bow and stern waves and between 0.7 and 1.3 of the linear natural roll period for synchronous roll in beam waves.

3.1.7.22 The simplifications considered so far reduce the total number of assessment cases (i.e. number of combinations of wave height, period and direction and ship forward speed) from about 200 (number of sea states with non-zero probabilities in a scatter table) times 19 (number of wave directions) times 7 (number of forward speeds), i.e. about 25000 altogether, to about 10 (the number of wave periods covering the ranges defined in paragraph 3.1.7.20).

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| Table 3.1.4 Ratio of wave encounter period (corresponding to mean zero-upcrossing period) leading to maximum failure rate to natural roll period at 0 and 40 degree roll amplitudes | | | | | | |
| Ship | GM, m | Tr, s, IS Code | Tr, s, from roll decay at roll amplitude of | |  |  |
| 0o | 40o |
| Parametric Roll in Bow Waves | | | | | | |
| 1700 TEU Container Ship | 0.50 | 30.3 | 29.3 |  | 0.346 |  |
| 1.20 | 19.6 | 19.4 | 19.4 | 0.427 | 0.427 |
| Cruise Vessel | 1.50 | 20.0 | 19.8 |  | 0.465 |  |
| 2.00 | 17.4 | 17.2 | 36.9 | 0.481 | 0.225 |
| 8400 TEU Container Ship | 0.89 | 36.7 | 36.7 | 28.8 | 0.301 | 0.384 |
| 1.26 | 31.4 | 31.3 | 27.2 | 0.353 | 0.406 |
| 2.01 | 25.9 | 25.7 | 23.8 | 0.358 | 0.387 |
| 14000 TEU Container Ship | 1.00 | 39.0 | 38.8 |  | 0.308 |  |
| 2.00 | 27.6 | 27.6 |  | 0.400 |  |
| 3.00 | 22.5 | 22.6 |  | 0.407 |  |
| Parametric Roll in Stern Waves | | | | | | |
| 1700 TEU Container Ship | 0.50 | 30.3 | 29.3 |  | 0.314 |  |
| 1.20 | 19.6 | 19.4 | 19.4 | 0.427 | 0.427 |
| Cruise Vessel | 1.50 | 20.0 | 19.8 |  | 0.465 |  |
| 8400 TEU Container Ship | 0.89 | 36.7 | 36.7 | 28.8 | 0.301 | 0.384 |
| 1.26 | 31.4 | 31.3 | 27.2 | 0.353 | 0.406 |
| 2.01 | 25.9 | 25.7 | 23.8 | 0.358 | 0.387 |
| 14000 TEU Container Ship | 1.00 | 39.0 | 38.8 |  | 0.308 |  |
| 2.00 | 27.6 | 27.6 |  | 0.400 |  |
| 3.00 | 22.5 | 22.6 |  | 0.448 |  |
| Synchronous Roll in Beam Waves | | | | | | |
| 1700 TEU Container Ship | 5.75 | 8.9 | 8.8 | 9.2 | 0.941 | 0.901 |
| 6.75 | 8.2 | 8.2 | 8.5 | 1.234 | 1.187 |
| RoPax Vessel | 3.70 | 11.7 | 11.8 | 15.5 | 0.780 | 0.594 |
| 5.20 | 9.9 | 9.8 | 12.1 | 0.939 | 0.762 |
| 5.90 | 9.3 | 9.4 | 11.0 | 0.979 | 0.837 |
| 6.60 | 8.8 | 9.0 | 10.2 | 1.022 | 0.903 |
| 8400 TEU Container Ship | 5.00 | 15.5 | 15.4 | 15.1 | 0.776 | 0.795 |
| 6.93 | 13.1 | 13.2 | 13.0 | 0.767 | 0.781 |
| 14000 TEU Container Ship | 9.00 | 13.0 | 13.0 | 12.9 | 0.849 | 0.854 |
| 12.00 | 11.3 | 11.4 | 11.2 | 0.888 | 0.905 |

3.1.7.23 For a given number of the required assessment cases, simulation (or model testing) time can also be reduced. For example, the extrapolation of failure rate over wave height can be used to reduce simulation time not only in the full probabilistic assessment but also in the design situations approach, when the required simulation time becomes too large.

3.1.7.24 One more possibility to reduce the computational or model testing time is to stop further realisations of the design sea state in numerical simulations or model tests once it is obvious that further realisations are not going to change the final conclusion, i.e. when the estimate of the lower boundary of, for example, a 95%-confidence interval of the mean time to failure exceeds the specified threshold (thus, the loading condition can be considered as allowed) or when the estimate of the upper boundary of the 95%-confidence interval of the mean time to failure is less than the specified threshold (thus, the loading condition can be considered as not allowed). Figure 14 shows simulation results for the 14000 TEU container ship in three loading conditions in a design sea state corresponding to parametric roll in bow waves: if the acceptance threshold for the mean time to stability failure is set to (only as an example) 102 s, the loading condition with GM=1.0 m can be considered as not allowed already after 80 simulations, the other one with GM=2.0 m will require 200 realisations, and the loading condition with GM=3.0 m can be considered as allowed already after about 20 simulations.

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| Figure 3.1.14 Simulation results for 14000 TEU container ship in design situation corresponding to parametric roll in bow waves. Left: time to stability failure from individual realisations for GM of 1.0 (+), 2.0 (▲) and 3.0 (○) m and estimates of mean time to failure (⎯, - - - and - ⋅ -, respectively). Right: estimates of mean time to failure (solid lines) and upper (- - -) and lower (- ⋅ -) boundaries of their 95%-confidence intervals for loading conditions with GM=1.0 m (black), 2.0 m (blue) and 3.0 m (red). |

**3.1.8 Non-probabilistic direct stability assessment**

3.1.8.1 A drawback of a probabilistic assessment is the need to encounter stability failure events in simulations or in model tests, which requires long durations of simulations or model tests for relevant cases. An appealing idea is to combine design situations with non-probabilistic criteria, e.g. mean maximum roll amplitude per given exposure time, mean roll amplitude etc., which require much less simulation or model testing time for their definition.

3.1.8.2 The idea is the same as was shown in Figure 3.1.4: as long as the selected non-probabilistic criterion is monotonously related to the true safety measure (e.g. long-term safety failure probability) and scatter between ships, loading conditions and forward speeds is not excessive, the simplified criterion can be directly used for norming, and its acceptance threshold can be defined directly using results of a non-probabilistic assessment for a sufficient number of representative sample cases.

3.1.8.3 In the documents SDC 4/5/8 and SDC 4/INF.8, this method was verified for roll in beam sea to address dead ship condition and excessive acceleration stability failures. Two non-probabilistic short-term criteria (mean roll amplitude and mean 3 hour maximum roll amplitude) were tested for different ships, loading conditions and forward speeds in irregular beam seaways. The latter criterion has shown significantly better results than the former one, therefore, it was used here in combination with design situations to develop a non-probabilistic direct assessment concept for parametric and synchronous roll.

3.1.8.4 The ships and loading conditions used are the same as listed in Table 3.1.1. In the first step, different forward speeds were evaluated separately. One wave direction per failure mode was used for design situations: 180, 0, 90 and 0 degree for parametric roll in bow and stern waves, synchronous roll in beam waves and pure loss of stability, respectively.

3.1.8.5 As in the previous section, sea states selected along the lines of constant density of seaway occurrence probability fs, Figure 3.1.5, were used as possible design sea states; as the simplified criterion s, maximum (over design sea states) of the mean 3 hour maximum roll amplitude was used.

3.1.8.6 To compute the mean 3 hour maximum roll amplitude, numerical simulations were performed in 50 realisations of each sea state by random variation of frequencies, directions and phases of components modelling seaway.

3.1.8.7 Evaluation of the mean 3 hour maximum roll amplitude is impossible in cases with capsizings, since then the roll amplitude is not defined. To distinguish such cases in plots, the mean 3 hour maximum roll amplitude is shown for them as 60 degree for ease of identification, since mean 3 hour maximum roll amplitude never achieved 60 degree in simulations where capsizings did not happen (for the considered ships).

3.1.8.8 Figure 3.1.15 to Figure 3.1.18 show the mean long-term stability failure rate w vs. the mean 3 hour maximum roll amplitude for parametric roll in bow (Figure 3.1.15) and stern (Figure 3.1.16) waves, synchronous roll in beam waves (Figure 3.1.17) and pure loss of stability (Figure 3.1.18). The shown mean 3 hour maximum roll amplitude is defined as maximum over all wave periods in design sea states with the density fs of occurrence probability of 10-7, 10-6, 10-5, 10-4, 10-3 and 10-2 (m⋅s)‑1 for wave directions 180 degree (for parametric roll in bow waves), 0 degree (parametric roll in stern waves) and 90 degree (synchronous roll in beam waves), and for combined conditions of wave direction 0 degree, encounter peak wave period more than 30 s and wave length equal to ship length (for pure loss of stability). Each point corresponds to one ship in one loading condition at one forward speed.

3.1.8.9 Correlation between the mean long-term stability failure rate and mean 3 hour maximum roll amplitude in design sea states is very poor, especially in cases with small roll motions. Although increasing roll motions significantly improve this correlation, they also lead to capsizings which make the evaluation of non-probabilistic criteria impossible.

3.1.8.10 To select forward speeds to be used in the assessment, the mean 3-hour maximum roll amplitude in head and following (for parametric roll), beam (synchronous roll) and following (pure loss of stability) waves in sea states with probability density 10-5 (m⋅s)‑1 is plotted vs. forward speed in Figure 3.1.19 to Figure 3.1.22. The results are similar to the speed dependency of the probabilistic criterion: for parametric roll in head waves and for synchronous roll, decreasing forward speed maximizes 3-hour maximum roll, whereas for pure loss of stability, the greatest roll responses correspond to the maximum forward speed. For parametric roll in following waves, maximum roll may both decrease or increase with increasing forward speed; however, for the most critical loading conditions, low forward speeds lead to maximum roll response. Therefore, similar recommendations can be given for the selection of forward speed as those in the probabilistic design situations approach.

**3.1.9 Influence of propulsion, steering and seakeeping**

3.1.9.1 For some stability failure modes, neglecting propulsion and steering abilities of a ship, as well as such seakeeping problems as excessive vertical motions and accelerations and excessive loads at high forward speeds in bow waves, can lead to non-conservative errors in design assessment or misleading operational recommendations. In particular,

.1 For pure loss of stability and surf-riding/broaching stability failures, which are relevant in stern waves, neglecting speed limitations does not lead to non-conservative errors, thus is not critical for dynamic stability.

.2 Dead ship condition stability failure is relevant only at zero forward speed in beam seaway, therefore these problems are also not critical.

.3 For excessive acceleration stability failure, achievable forward speed in beam seaway rather moderately influences roll motion (due to decreasing roll damping with decreasing forward speed) and does not influence the design assessment (which is performed at zero forward speed).

.4 For parametric roll in bow waves, neglecting propulsion, steering and seakeeping abilities can lead to over-estimation of ship’s safety in the design assessment, when safe but unattainable combinations of the ship’s speed and course are weighted as possible.

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| Figure 3.1.15 Parametric roll in bow waves: mean long-term failure rate w(ship,LC,vs), 1/s, y axis, in wave directions from 170 to 180 degree (left), 160 to 180 degree (middle) and 150 to 180 degree (right) vs. non-probabilistic criterion, degree, x axis – mean 3-hour maximum roll amplitude in head waves, maximum over design sea states along lines with sea state occurrence probability density fs equal to (from top to bottom) 10-7, 10-6, 10-5, 10-4, 10-3 and 10-2 (m⋅s)‑1. Each point corresponds to one ship (different symbols), one loading condition and one forward speed. Points with mean 3-hour maximum roll amplitude equal to 60 degree indicate cases with capsizings. |

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| Figure 3.1.16 Parametric roll in stern waves: mean long-term stability failure rate w(ship,LC,vs), 1/s, y axis, in wave directions from 0 to 10 degree (left), 0 to 20 degree (middle) and 0 to 30 degree (right) vs. non-probabilistic criterion, degree, x axis – mean 3-hour maximum roll amplitude in following waves, maximum over design sea states along lines with sea state occurrence probability density fs equal to (from top to bottom) 10-7, 10-6, 10-5, 10-4, 10-3 and 10-2 (m⋅s)‑1. Each point corresponds to one ship (different symbols), one loading condition and one forward speed. Points with mean 3-hour maximum roll amplitude equal to 60 degree indicate cases with capsizings. |

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| Figure 3.1.17. Synchronous roll in beam waves: mean long-term stability failure rate w(ship,LC,vs), 1/s, y axis, in wave directions from 80 to 100 degree (left), 70 to 110 degree (middle) and 60 to 120 degree (right) vs. non-probabilistic criterion, degree, x axis – mean 3-hour maximum roll amplitude at μ=90 degree, maximum over design sea states along lines with sea state occurrence probability density fs equal to (from top to bottom) 10-7, 10-6, 10-5, 10-4, 10-3 and 10-2 (m⋅s)‑1. Each point corresponds to one ship (different symbols), one loading condition and one forward speed. Points with mean 3-hour maximum roll amplitude equal to 60 degree indicate cases with capsizings. |

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| Figure 3.1.18. Pure loss of stability in following waves: mean long-term stability failure rate w(ship,LC,vs), 1/s, y axis, vs. non-probabilistic criterion, degree, x axis – mean 3-hour maximum roll amplitude in following waves, maximum over design sea states along lines with sea state occurrence probability density fs equal to (from left to right, top then bottom) 10-7, 10-6, 10-5, 10-4, 10-3 and 10-2 (m⋅s)‑1. Each point corresponds to one ship (different symbols), one loading condition and one forward speed. Points with mean 3-hour maximum roll amplitude equal to 60 degree indicate cases with capsizings. |

3.1.9.2 To estimate the influence of propulsion ability on parametric roll in head waves, average (over all significant wave heights and wave periods) rate of parametric roll stability failures in head waves was calculated with and without considering maximum attainable speed in head waves. In both cases, the forward speed was applied that minimises the stability failure rate, but in the calculations considering propulsion ability, the range of speeds was restricted by the condition that the required engine power should not exceed the available power. Figure 23 shows the result as the rate of stability failures considering speed limit plotted depending on the rate of stability failures without considering speed limit.

3.1.9.3 The results show that the rate of stability failures increases by several orders of magnitude if propulsion ability is considered. This means that assessment at zero forward speed in head waves (already proposed in the design situations method using other considerations) is a conservative but realistic assumption.

**3.1.10 Definition of standard and thresholds**

3.1.10.1 To distinguish allowed and not allowed loading conditions, an acceptance standard should be defined for the mean long-term stability failure rate w, as well as coherent short-term acceptance thresholds for the criteria used in the simplified assessment procedures (i.e. for the mean short-term stability failure rate r and for the mean three-hour maximum roll amplitude 3h in design situations) for all stability failure modes.

3.1.10.2 The relationship between the long-term probabilistic criterion w, on the one hand, and the short-term design-situation criteria r (Figure 3.1.6, Figure 3.1.7, Figure 3.1.8 and Figure 3.1.9) and 3h (Figure 3.1.15, Figure 3.1.16, Figure 3.1.17 and Figure 3.1.18), on the other hand, are universal as pure statistical relationships and do not depend on the code; besides, the dependencies between w and r are sharp, thus they are not subject to stochastic noise. Thus, short-term acceptance thresholds for r and 3h can be defined directly from case studies in design situations, without the need for a full probabilistic assessment, and the standard for w for the full probabilistic assessment can be derived from the short-term design-situation thresholds for r and 3h.

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| Figure 3.1.19 Mean 3-hour maximum roll amplitude due to parametric roll in head waves in sea states with fs=10-5 (m⋅s)‑1 vs. forward speed. Each plot corresponds to one ship; different symbols correspond to different loading conditions. | Figure 3.1.20 Mean 3-hour maximum roll amplitude due to parametric roll in following waves in sea states with fs=10‑5 (m⋅s)‑1 vs. forward speed. Each plot corresponds to one ship; different symbols correspond to different loading conditions. |

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| Figure 3.1.21 Mean 3-hour maximum roll amplitude due to synchronous roll in beam waves in sea states with fs=10-5 (m⋅s)‑1 vs. forward speed; each plot corresponds to one ship; different symbols correspond to different loading conditions |

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| Figure 3.1.22 Mean 3-hour maximum roll amplitude due to pure loss of stability in following waves in sea states with fs=10‑5 (m⋅s)‑1 vs. forward speed. Each plot corresponds to one ship; different symbols correspond to different loading conditions. |

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|  | Figure 3.1.23 Rate of parametric roll stability failures in head waves considering (y axis) and not considering (x axis) attainable forward speed for three container ships (different symbols) in three loading conditions each |

3.1.10.3 However, the missing link is the relationship between the standard for the mean long-term stability failure rate in the full probabilistic assessment and the mean stability failure rate in the real operation, i.e. the actual safety level: this relationship is uncertain, whereas the stability failure rate in the full probabilistic assessment may differ from the failure rate in real operation by few orders of magnitude due to several factors:

.1 full probabilistic assessment is conducted in the rather severe North Atlantic wave climate and the mean safety level relates to the world-wide operation;

.2 routing and heavy-weather avoidance are not considered;

.3 assessment is performed separately for each loading condition, thus the worst loading condition (which may never occur in practice) has the dominating effect on the results;

.4 unsafe forward speeds and courses, avoided in reality in heavy weather, produce dominating (by few orders of magnitude) contributions to the long-term failure rate. For example, principal parametric resonance in following waves, especially at low speeds, provides dominating contributions to failure rate for loading conditions with low GM, Figure 3.1.24, whereas in reality such situations are avoided (stern slamming, low free board) or impossible (inability to keep course).

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| Figure 3.1.24. Contributions to mean long-term stability failure rate w (1/s, y-axis) from principal parametric resonance in bow (left) and stern (right) waves (1/s, x-axis); symbol types and colours differentiate ships and loading conditions | |

3.1.10.4 To estimate the lower and upper bounds for the standard for the mean long-term stability failure rate w in the full probabilistic assessment and the short-term threshold for the stability failure rate r in design situations, the following considerations were applied:

.1 In the Appendix to the proposal for the Guidelines for operational measures, the value 2.64⋅10‑3 accidents per ship per year was proposed as the required safety level, based on FSA studies for container vessels, LNG carriers, crude oil tankers, cruise ships, RoPax and general cargo vessels (ref. documents MSC 83/INF.8, MSC 83/INF.3, MEPC 58/INF.2, MSC 85/INF.2, MSC 85/INF.3 and MSC 88/INF.8, respectively). From this figure, a conservative estimate of the standard for the full probabilistic assessment in the North Atlantic wave climate was defined as 2.6⋅10‑8 1/s considering factors mentioned in paragraphs 3.1.10.3.1 to 3.1.10.3.4, which is used here as one of estimates for the lower bound of the standard for w;

.2 It is useful to note that a similar study on setting standards for the vertical bending moment[[3]](#footnote-3) has shown that the definition of the standard for results of numerical assessment as once per design life in the North Atlantic wave climate leads to too conservative results compared to the existing fleet (known to be sufficiently safe) and requirements of classification societies; to harmonise the results of direct assessment with classification rules, a “routing factor” 0.85 was proposed, with which wave heights should be multiplied. For comparison, the present results of the full probabilistic assessment were reevaluated with 0.85-scaled wave heights, which leads to the standard for w of 1.4⋅10‑8 1/s; since this value is close to the estimate of the standard in paragraph 3.1.10.4.1, it was not used;

.3 The results of assessment with respect to the dead ship stability failure mode in design situations were sorted, suggesting that loading conditions satisfying the Weather Criterion of the 2008 Intact Stability Code should also satisfy the direct stability assessment with respect to the dead ship stability failure mode, whereas loading conditions failing the Weather Criterion should also fail the direct assessment with respect to the dead ship condition stability failure mode; this led to the upper and lower estimates for the short-term design-situation threshold for r shown in Table 3.1.5.

.4 Stability failure rate defined from numerical simulations or, especially, model tests cannot be too high or too low: in the former case, it will be difficult to minimize the influence of the initial conditions (thus, it is proposed to limit the mean stability failure rate in design situations as r≤10‑3 1/s, i.e. one failure in 103 s), and in the latter case, required testing or simulation time will be too large (thus, it is proposed to limit the mean stability failure rate in design situations as r≥10‑4 1/s, i.e. one failure in 3 hours).

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| Table 3.1.5. Estimates of lower and upper boundaries for short-term design-situation r-threshold from comparison with Weather Criterion | | | | | | |
| fs, (m⋅s)‑1 | 10‑2 | 10‑3 | 10‑4 | 10‑5 | 10‑6 | 10‑7 |
| lower | 1.8⋅10‑34 | 1.0⋅10‑15 | 2.8⋅10‑10 | 7.5⋅10‑8 | 1.8⋅10‑6 | 1.4⋅10‑5 |
| upper | 1.7⋅10‑9 | 2.8⋅10‑5 | 4.7⋅10‑4 | 1.7⋅10‑3 | 3.5⋅10‑3 | 5.6⋅10‑3 |

3.1.10.5 Figure 3.1.25 plots together the w(r) dependencies of the mean long-term stability failure rate w on the mean short-term stability failure rate in design situations r for all stability failure modes from Figure 3.1.6, Figure 3.1.7, Figure 3.1.8 and Figure 9 for fs=10‑2 to 10‑7 (m⋅s)‑1 together with the estimates for the bounds for w-standard and r-threshold according to the considerations in paragraphs 3.1.10.4.1, 3.1.10.4.3 and 3.1.10.4.4; the bounds for w-standard are transferred into bounds for r-threshold and the other way around using the w(r) dependencies. The colours of the resulting rectangles indicate:

.1 red: requirements according to FSA studies, paragraph 3.1.10.4.1;

.2 green: Weather Criterion results, paragraph 3.1.10.4.3 and Table 3.1.5;

.3 blue: practical considerations in paragraph 3.1.10.4.4.

3.1.10.6 Overlapping areas, indicated with arrows in Figure 25 and shown in increased resolution in Figure 3.1.26, mean the possibility of non-contradicting combination of all estimates and show that the direct assessment using design situations is possible in design sea states with probability density fs=10‑5, 10‑6 and, marginally, 10‑7 (m⋅s)‑1, as well as for all intermediate values of fs. For greater or lower values of fs, the areas corresponding to various estimates do not overlap, note, however, that for design sea states with the probability density 10‑4 (m⋅s)‑1, the limitation is long simulation time, which is not a crucial problem for some numerical methods and, besides, that the required simulation time reduces if the standard for the mean long-term stability failure rate w increases.

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| Figure 3.1.25 Combined dependencies w(r) of mean long-term stability failure rate w on mean short-term design-situation stability failure rate r for all stability failure modes from Figure 3.1.6, Figure 3.1.7, Figure 3.1.8 and Figure 3.1.9 for fs of (from top left to bottom right) 10‑2 to 10‑7 (m⋅s)‑1 together with estimates according to paragraphs 3.1.10.4.1, 3.1.10.4.3 and 3.1.10.4.4; arrows indicate overlapping areas, where non-contradicting combination of estimates is possible |

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| Figure 3.1.26 Definition of standard and threshold (increased resolution plots from Figure 3.1.25); thick line rectangle indicates overlapping area, circle indicates selection |

3.1.10.7 According to the analysis in Figure 3.1.25 and Figure 3.1.26, the value 2.6⋅10‑8 1/s seems suitable as a conservative estimate for the standard for the mean long-term stability failure rate w in the full probabilistic assessment. As thresholds for the mean short-term stability failure rate r in design situations, the following values are suitable:

.1 one stability failure in 20 hours in design sea states with fs=10‑4 (m⋅s)‑1;

.2 one stability failure in 3 hours (slightly conservative) to one stability failure in 2 hours in design sea states with fs=10‑5 (m⋅s)‑1;

.3 one stability failure in one hour (rather conservative) to one stability failure in 40 minutes in design sea states with fs=10‑6 (m⋅s)‑1;

.4 one stability failure in 15 minutes in design sea states with fs=10‑7 (m⋅s)‑1; however, such a high failure rate may lead to a significant influence of initial conditions, and, besides, sea states corresponding to fs=10‑7 (m⋅s)‑1 may be too steep for model tests.

3.1.10.8 Table 3.1.6 shows the significant wave height vs. the mean zero-upcrossing wave period at fs=10‑5 and 10‑6 (m⋅s)‑1 for unrestricted service, i.e. wave scatter table from IACS Rec. 34.

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| Table 3.1.6. Significant wave height depending on mean zero-upcrossing wave period for sea states with density of occurrence probability of 10-5 and 10-6 (m⋅s)‑1 | | | | | | | | | | | | | | |
| fs, (ms)‑1 | Tz, s | | | | | | | | | | | | | |
| 4.5 | 5.5 | 6.5 | 7.5 | 8.5 | 9.5 | 10.5 | 11.5 | 12.5 | 13.5 | 14.5 | 15.5 | 16.5 | 17.5 |
|  | | | | | | | | | | | | | | |
| 10‑5 | 2.8 | 5.5 | 8.2 | 10.6 | 12.5 | 13.8 | 14.6 | 15.1 | 15.1 | 14.8 | 14.1 | 12.9 | 10.9 | - |
| 10‑6 | 3.7 | 6.8 | 9.8 | 12.3 | 14.3 | 15.7 | 16.6 | 17.1 | 17.3 | 17.2 | 16.7 | 15.9 | 14.7 | 12.9 |

3.1.10.9 Whereas the threshold for the mean 3-hour maximum roll amplitude 3h for non-probabilistic assessment in design situations can be defined in a similar way, a slightly different approach was used:

.1 the threshold for 3h was set to half of the heel angle in the definition of the stability failure to avoid capsizings in the relevant model tests or numerical simulations;

.2 the maximum value of the mean long-term stability failure rate w, computed in the full probabilistic assessment, was found over all ships, loading conditions and forward speeds satisfying the chosen 3h-threshold in design situations;

.3 in this way, the maximum value of the mean long-term stability failure rate w becomes a function of the probability density fs defining design sea states, in which the non-probabilistic assessment is performed.

3.1.10.10 Figure 3.1.27 plots the resulting mean long-term stability failure rate w (y-axis) as a function of the probability density fs defining design sea states (x axis). To satisfy the selected standard for the mean long-term stability failure rate 2.6⋅10‑8 1/s (dashed line), the design sea states with the probability density fs=7⋅10‑5 (m⋅s)‑1 (circle) should be used.

3.1.10.11 Table 3.1.7 shows the significant wave height vs. the mean zero-upcrossing wave period for fs=7⋅10‑5 (m⋅s)‑1 for unrestricted service (IACS Rec. 34 wave scatter table).

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|  | Figure 3.1.27 Definition of design sea states for non-probabilistic assessment |

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| Table 3.1.7 Significant wave height vs. mean zero-upcrossing wave period for sea state probability density of 7⋅10‑5 (m⋅s)‑1 in North Atlantic wave climate | | | | | | | | | | | | |
| Tz,s | 4.5 | 5.5 | 6.5 | 7.5 | 8.5 | 9.5 | 10.5 | 11.5 | 12.5 | 13.5 | 14.5 | 15.5 |
| hs,m | 2.0 | 4.4 | 6.9 | 9.1 | 10.9 | 12.1 | 12.8 | 13.1 | 13.0 | 12.5 | 11.3 | 9.0 |

3.1.10.12 The proposed standard for the mean long-term stability failure rate w and thresholds for mean short-term failure rate r and mean 3 hour maximum roll amplitude 3h in design situations can be fine-tuned using either the full probabilistic assessment or assessment in design situations for accidental ships in accidental loading conditions and applying Figure 3.1.25, Figure 3.1.26 and Figure 3.1.27 to scale the long-term standard into short-term threshold or vice versa.

**3.2 Detailed description of direct counting for full probabilistic direct stability assessment procedure**

3.2.1 The detailed description of direct counting for full probabilistic direct stability assessment procedure in 3.1.3.5.4 is shown here.

3.2.2 The direct counting procedure is based on the simulation of ship motions in multiple independent realisations of the same irregular seaway and counting the number of stability failures. The procedure provides the estimate of the upper boundary of the 95%-confidence interval of the rate of stability failures, which is required in the probabilistic DSA in design situations, in the full probabilistic DSA and in OM.

3.2.3 The sea state is assumed to be modelled as a sum of harmonic components, , where,  are amplitudes, ωi frequencies, μi directions and εi phases of harmonic components, Sζζ is the wave energy spectrum and D is the wave energy angular spreading function. To generate independent realisations of the same irregular seaway, for each realisation the phases εi should be randomly selected in the interval [0,2π), frequencies ωi and directions μi can be assumed constant or randomly selected within the ranges Δωi and Δμi, respectively, and the amplitudes ai can be assumed constant. To generate random values, pseudo-random number generators can be used.

3.2.4 To ensure that stability failures satisfy the requirements to a stationary Poisson process, the procedure should avoid

.1 self-repetition effects by limiting the maximum duration of each simulation. The absence of self-repetition effects should be verified, e.g. using quantile plots of the sample mean time to failure vs. the number of failures. The maximum duration of 3 hours can be used if not less than 103 frequencies per wave direction are used for discretisation of wave energy spectrum;

.2 transient hydrodynamic effects at the beginning of simulations by switching off the counter of stability failures and the simulation timer during initial transients. The duration of transients can be assumed 50 roll periods; and

.3 auto-correlation of big roll motions. Although this effect over-estimates the sample mean failure rate, it also under-estimates the size of the confidence interval, thus it should be considered. As a solution, stopping a simulation after the first failure can be used.

3.2.5 If a stability failure is encountered during a simulation, the simulation is stopped, the total number of failures N is increased by 1 and the total simulation time tt is increased by the time to failure, otherwise N is not increased and tt is increased by the simulation time. With the updated N and tt values, the sample mean time to failure and the maximum likelihood estimate of the failure rate are updated as  and , respectively, and the upper boundary of the 95%-confidence interval of failure rate is calculated as .

3.2.6 If the stability failure rate is low, the required number of simulations until the first stability failure may be too large, which is unpractical since the absence of stability failure during a long time should be sufficient for acceptance. To address such cases (for the first as well as following simulations), introduce variable N\* which is set to N+1 after each simulation, i.e. also those in which a stability failure did not happen, and calculate conservative estimates ,  and . Since =rU after stability failures and is a conservative estimate of rU after simulations where a stability failure did not happen, calculation of rU is not required and  can be used for *acceptance check* in all cases (e.g. in probabilistic direct assessment in design situations, once this value it is less than the acceptance standard, further simulations are not required and the loading condition can be considered as acceptable in the considered situation).

3.2.7 To avoid unnecessary simulations, also the lower boundary of the 95%-confidence interval of the failure rate is estimated as  for not acceptance check (e.g. in probabilistic direct assessment in design situations, once rL exceeds the standard in at least one design situation, the loading condition can be considered as not acceptable without further simulations or tests).

3.2.8 Note that the functions  and  are available in many software packages; in MS Excel, they can be calculated as chisq.inv.rt(α/2;2\*N) and chisq.inv(α/2;2\*N), respectively.

3.2.9 Short summary of the direct counting procedure:

.1 the procedure is based on numerical simulations or model tests in multiple independent realisations of the same irregular seaway and counting the number of stability failures. Such realisations can be generated by random variation of phases (and, possibly, frequencies, directions and amplitudes) of harmonic waves discretising the wave energy spectrum. Numerical simulations or model tests in these realisations are carried out for arbitrary simulation time, which is limited by the maximum duration and first stability failure. During initial transients, the counter of stability failures and the simulation timer are switched off;

.2 After each numerical simulation or model test, the number of stability failures ΔN (1 if simulation ended with a stability failure and 0 otherwise) and duration of simulation Δt are recorded; then N\* is calculated as N+1, the total number of failures N is increased by ΔN and the total simulation time tt is increased by Δt. The sample mean time to failure is calculated as , the maximum likelihood estimate of the failure rate as  and their conservative estimates as  and , respectively;

.3 For *acceptance* check, the upper boundary of the 95%-confidence interval of the failure rate  is conservatively estimated as ; for *not acceptance* check, the lower boundary of the 95%-confidence interval of the failure rate rL is estimated as .

**3.3 Detailed description of direct counting for probabilistic procedure in design situations**

3.3.1 The detailed description of direct counting for probabilistic direct stability assessment procedure in design situations in 3.5.4 of the Guidelines is shown here.

3.3.2 For the probabilistic direct stability assessment in design situations, statistical extrapolation is not required, therefore the proposed procedure is based on direct counting.

3.3.3 The procedure performs an *acceptance check*, requiring that the upper boundary rU of the 95%-confidence interval of the failure rate does not exceed the standard λ in all design sea states. To save simulation or testing time, the procedure also introduces a *not acceptance check*, considering a loading condition as not acceptable once the lower boundary rL of the 95%-confidence interval of failure rate exceeds the standard in at least one design situation.

3.3.4 The procedure is based on numerical simulations or model tests in multiple independent realisations of the same irregular seaway and counting the number of stability failures. The sea state is assumed to be modelled as a sum  of harmonic components with amplitudes ai, frequencies ωi, directions μi and phases εi. To generate such realisations, the phases εi should be randomly selected in the interval [0,2π), whereas frequencies ωi and directions μi can be either assumed constant or randomly selected within ranges Δωi and Δμi, respectively. To generate random values, pseudo-random number generators can be used.

3.3.5 Since stability failures should satisfy a Poisson process assumptions, procedure addresses self-repetition effects by limiting maximum simulation time in each realisation, transient hydrodynamic effects at the beginning of simulations by switching off the counter of stability failures and simulation timer during the initial 50 roll periods and auto-correlation of big roll motions by stopping a simulation after the first stability failure; apart from limited duration and stopping after the first stability failure, duration of each simulation is arbitrary.

3.3.6 The *acceptance* requirement rU < λ becomes, using the equation  for the upper boundary of the 95%-confidence interval of the failure rate with α=0.05 and definition ,

|  |  |
| --- | --- |
|  | (3.3.1) |

3.3.7 Similarly, the *not acceptance* requirement rL < λ leads, using the equation for the lower boundary of the 95%-confidence interval of the failure rate  with α=0.05 and definition , to

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| --- | --- |
|  | (3.3.2) |

3.3.8 The acceptance check (1) is not convenient in practice since it requires encountering at least one stability failure, although the time until the first failure may be much too long for acceptable loading conditions and even for not acceptable ones away from resonance. On the other hand, a sufficiently long simulation time without stability failure can itself be used for acceptance: conservatively assuming N = 1 and thus,  in eq. (1) yields that a simulation can be stopped (with the *acceptance* decision) when the simulation time without failure is

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|  | (3.3.3) |

3.3.9 To extend this on the second and further failures, introduce  in eq. (3.3.1) ( is the sample mean time to failure from the previous N − 1 stability failures) and assume, conservatively, TN=t; this yields that a simulation can be stopped, with the *acceptance* decision in the considered design situation, when the simulation time without failure achieves

|  |  |
| --- | --- |
|  | (3.3.4) |

3.3.10 Note that eq. (3.3.4) automatically reduces to eq. (3.3.3) when N = 1, therefore only eq. (4) is required for the acceptance check for the first and further simulations.

3.3.11 , the quantile function corresponding to a lower tail area equal *p*, of the χ2-distribution with *f* degrees of freedom, is available in many software packages. In MS Excel,  can be calculated as chisq.inv.rt(α/2;2\*N) and  as chisq.inv(α/2;2\*N).

3.3.12 A short summary of the procedure for probabilistic DSA in design situations:

.1 the procedure is based on numerical simulations or model tests in multiple independent realisations of the same irregular seaway. Such realisations can be generated by random variation of phases (and, possibly, frequencies, directions and amplitudes) of harmonic waves discretising the wave energy spectrum. Simulations or tests can be carried out for arbitrary time until the maximum duration or first stability failure. The counter of stability failures and the simulation timer are switched off during initial transients;

.2 after each simulation or test, the total number of failures N is increased by ΔN (1 if ended with a stability failure and 0 otherwise), the total simulation time tt is increased by the duration of simulation Δt and the sample mean time to failure is updated as ;

.3 if a simulation achieves time  without stability failure, with , the loading condition can be considered *acceptable* in the considered design situation without further simulations;

.4 if the sample mean time to failure after a stability failure satisfies condition , where  and , the loading condition can be considered as *not acceptable* without further simulations.

**3.4 Application example of direct counting method for full probabilistic procedure**

3.4.1 This is an application example of the direct counting procedure described in Annex 2. The direct counting procedure is based on the simulation of ship motions in multiple independent realisations of an irregular seaway and counting the number of stability failures. The procedure provides the estimate of the upper boundary of the 95%-confidence interval of the rate of stability failures, which is required in the probabilistic DSA in design situations, in the full probabilistic DSA and in OM.

3.4.2 The sea state is modelled as a sum  of harmonic components with amplitudes , frequencies ωi, directions μi and phases εi, using 19 wave directions and 103 components per wave direction. The Bretschneider wave energy spectrum Sζζ and the cos2-wave energy angular spreading function D are used. For each realisation, the phases εi are randomly selected in the interval [0,2π), frequencies ωi and directions μi are randomly selected within their ranges Δωi and Δμi, respectively, and the amplitudes ai are equal for all components. To generate random values, pseudo-random number generator was used, applying the internal computer timer as a seed number.

3.4.3 To avoid self-repetition effects, the duration of each simulation is limited by 1 hour. Transient hydrodynamic effects at the beginning of simulations are avoided by switching off the counter of stability failures and simulation timer during the initial 50 roll periods. To avoid the effects of auto-correlation of big roll motions, each simulation was stopped after the first encountered failure.

3.4.4 After each simulation, the following parameters were recorded: the number of realisation i, the number of stability failures in the simulation ΔN (1 if the simulation ended with a stability failure and 0 otherwise) and duration of simulation Δt (time to failure if realisation ended with a stability failure and 1 hour otherwise).

3.4.5 Based on these parameters, N\* was calculated as N+1, the total number of failures N was increased by ΔN, and the total simulation time tt was increased by Δt. The sample mean time to failure and the maximum likelihood estimate of the failure rate were calculated as  and , respectively, and their conservative estimates as  and , respectively.

3.4.6 For acceptance check, the conservative estimate of the upper boundary of the 95%-confidence interval of the failure rate was calculated as ; for not acceptance check, the lower boundary of the 95%-confidence interval of the failure rate was estimated as . The functions  and  were calculated with MS Excel as chisq.inv.rt(0.05/2;2\*N) and chisq.inv(0.05/2;2\*N), respectively.

3.4.7 Table 1 shows example of results according to this procedure for parametric resonance in head seaway with a probability density fs=10‑5 (m⋅s)‑1 in the North Atlantic wave climate of a 1700 TEU container vessel with GM=1.8 m; Fig. 3.4.1 shows  and rL, which are required for acceptance and not acceptance checks, respectively, depending on the number of simulations i.

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| Table 3.4.1 Example results of direct counting procedure | | | | | | | | | | | |
| i | N | t,s | N\* | N | tt,s | ,s | ,1/s | ,s | ,1/s | ,1/s | rL,1/s |
| 1 | 1 | 1682.0 | 1 | 1 | 1682.0 | 1682.0 | 5.945e-4 | 1682.0 | 5.945e-4 | 2.193e-3 | 1.505e-5 |
| 2 | 0 | 3600.0 | 2 | 1 | 5282.0 | 5282.0 | 1.893e-4 | 2641.0 | 3.786e-4 | 1.055e-3 | 4.793e-6 |
| 3 | 1 | 2270.0 | 2 | 2 | 7552.0 | 3776.0 | 2.648e-4 | 3776.0 | 2.648e-4 | 7.378e-4 | 3.207e-5 |
| 4 | 0 | 3600.0 | 3 | 2 | 11152.0 | 5576.0 | 1.793e-4 | 3717.3 | 2.690e-4 | 6.478e-4 | 2.172e-5 |
| 5 | 0 | 3600.0 | 3 | 2 | 14752.0 | 7376.0 | 1.356e-4 | 4917.3 | 2.034e-4 | 4.897e-4 | 1.642e-5 |
| 6 | 1 | 1025.5 | 3 | 3 | 15777.5 | 5259.2 | 1.901e-4 | 5259.2 | 1.901e-4 | 4.579e-4 | 3.921e-5 |
| 7 | 1 | 2129.5 | 4 | 4 | 17907.0 | 4476.8 | 2.234e-4 | 4476.8 | 2.234e-4 | 4.896e-4 | 6.086e-5 |
| 8 | 1 | 1111.5 | 5 | 5 | 19018.5 | 3803.7 | 2.629e-4 | 3803.7 | 2.629e-4 | 5.385e-4 | 8.536e-5 |
| 9 | 0 | 3600.0 | 6 | 5 | 22618.5 | 4523.7 | 2.211e-4 | 3769.8 | 2.653e-4 | 5.159e-4 | 7.178e-5 |
| 10 | 0 | 3600.0 | 6 | 5 | 26218.5 | 5243.7 | 1.907e-4 | 4369.8 | 2.288e-4 | 4.450e-4 | 6.192e-5 |
| 11 | 0 | 3600.0 | 6 | 5 | 29818.5 | 5963.7 | 1.677e-4 | 4969.8 | 2.012e-4 | 3.913e-4 | 5.445e-5 |
| 12 | 0 | 3600.0 | 6 | 5 | 33418.5 | 6683.7 | 1.496e-4 | 5569.8 | 1.795e-4 | 3.492e-4 | 4.858e-5 |
| 13 | 0 | 3600.0 | 6 | 5 | 37018.5 | 7403.7 | 1.351e-4 | 6169.8 | 1.621e-4 | 3.152e-4 | 4.386e-5 |
| 14 | 0 | 3600.0 | 6 | 5 | 40618.5 | 8123.7 | 1.231e-4 | 6769.8 | 1.477e-4 | 2.873e-4 | 3.997e-5 |
| 15 | 0 | 3600.0 | 6 | 5 | 44218.5 | 8843.7 | 1.131e-4 | 7369.8 | 1.357e-4 | 2.639e-4 | 3.672e-5 |
| 16 | 1 | 136.5 | 6 | 6 | 44355.0 | 7392.5 | 1.353e-4 | 7392.5 | 1.353e-4 | 2.631e-4 | 4.964e-5 |
| 17 | 0 | 3600.0 | 7 | 6 | 47955.0 | 7992.5 | 1.251e-4 | 6850.7 | 1.460e-4 | 2.723e-4 | 4.592e-5 |
| 18 | 0 | 3600.0 | 7 | 6 | 51555.0 | 8592.5 | 1.164e-4 | 7365.0 | 1.358e-4 | 2.533e-4 | 4.271e-5 |
| 19 | 0 | 3600.0 | 7 | 6 | 55155.0 | 9192.5 | 1.088e-4 | 7879.3 | 1.269e-4 | 2.368e-4 | 3.992e-5 |
| 20 | 0 | 3600.0 | 7 | 6 | 58755.0 | 9792.5 | 1.021e-4 | 8393.6 | 1.191e-4 | 2.223e-4 | 3.748e-5 |
| 21 | 1 | 2235.5 | 7 | 7 | 60990.5 | 8712.9 | 1.148e-4 | 8712.9 | 1.148e-4 | 2.141e-4 | 4.614e-5 |
| 22 | 1 | 3505.0 | 8 | 8 | 64495.5 | 8061.9 | 1.240e-4 | 8061.9 | 1.240e-4 | 2.236e-4 | 5.355e-5 |
| 23 | 1 | 261.5 | 9 | 9 | 64757.0 | 7195.2 | 1.390e-4 | 7195.2 | 1.390e-4 | 2.434e-4 | 6.355e-5 |
| 24 | 1 | 1969.5 | 10 | 10 | 66726.5 | 6672.7 | 1.499e-4 | 6672.7 | 1.499e-4 | 2.560e-4 | 7.187e-5 |
| 25 | 0 | 3600.0 | 11 | 10 | 70326.5 | 7032.7 | 1.422e-4 | 6393.3 | 1.564e-4 | 2.615e-4 | 6.819e-5 |
| 26 | 0 | 3600.0 | 11 | 10 | 73926.5 | 7392.7 | 1.353e-4 | 6720.6 | 1.488e-4 | 2.488e-4 | 6.487e-5 |
| 27 | 1 | 1275.0 | 11 | 11 | 75201.5 | 6836.5 | 1.463e-4 | 6836.5 | 1.463e-4 | 2.445e-4 | 7.302e-5 |
| 28 | 0 | 3600.0 | 12 | 11 | 78801.5 | 7163.8 | 1.396e-4 | 6566.8 | 1.523e-4 | 2.498e-4 | 6.968e-5 |
| 29 | 1 | 2710.5 | 12 | 12 | 81512.0 | 6792.7 | 1.472e-4 | 6792.7 | 1.472e-4 | 2.415e-4 | 7.607e-5 |
| 30 | 1 | 1919.0 | 13 | 13 | 83431.0 | 6417.8 | 1.558e-4 | 6417.8 | 1.558e-4 | 2.512e-4 | 8.297e-5 |
| 31 | 1 | 3445.0 | 14 | 14 | 86876.0 | 6205.4 | 1.611e-4 | 6205.4 | 1.611e-4 | 2.559e-4 | 8.810e-5 |
| 32 | 0 | 3600.0 | 15 | 14 | 90476.0 | 6462.6 | 1.547e-4 | 6031.7 | 1.658e-4 | 2.596e-4 | 8.460e-5 |
| 33 | 1 | 1884.5 | 15 | 15 | 92360.5 | 6157.4 | 1.624e-4 | 6157.4 | 1.624e-4 | 2.543e-4 | 9.090e-5 |
| 34 | 0 | 3600.0 | 16 | 15 | 95960.5 | 6397.4 | 1.563e-4 | 5997.5 | 1.667e-4 | 2.578e-4 | 8.749e-5 |
| 35 | 1 | 634.0 | 16 | 16 | 96594.5 | 6037.2 | 1.656e-4 | 6037.2 | 1.656e-4 | 2.561e-4 | 9.468e-5 |
| 36 | 0 | 3600.0 | 17 | 16 | 100194.5 | 6262.2 | 1.597e-4 | 5893.8 | 1.697e-4 | 2.593e-4 | 9.128e-5 |
| 37 | 0 | 3600.0 | 17 | 16 | 103794.5 | 6487.2 | 1.542e-4 | 6105.6 | 1.638e-4 | 2.503e-4 | 8.811e-5 |
| 38 | 0 | 3600.0 | 17 | 16 | 107394.5 | 6712.2 | 1.490e-4 | 6317.3 | 1.583e-4 | 2.419e-4 | 8.516e-5 |
| 39 | 0 | 3600.0 | 17 | 16 | 110994.5 | 6937.2 | 1.442e-4 | 6529.1 | 1.532e-4 | 2.341e-4 | 8.239e-5 |
| 40 | 1 | 1801.5 | 17 | 17 | 112796.0 | 6635.1 | 1.507e-4 | 6635.1 | 1.507e-4 | 2.304e-4 | 8.780e-5 |

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|  |
| Fig. 3.4.1 Example results of direct counting procedure |

**3.5 Application example of direct counting for probabilistic procedure in design situations**

3.5.1 This is an application examples of probabilistic DSA in design situations for parametric roll stability failure mode in design situations.

3.5.2 The assessment is based on numerical simulation of ship motions in multiple independent realisations of irregular seaway and counting the number of stability failures. The sea state was modelled as a sum  of harmonic components with amplitudes ai={2Sζζ(ωi)D(μi)ΔωiΔμi}1/2, frequencies ωi, directions μi and phases εi; Sζζ is the Bretschneider wave energy spectrum and D is the cos2-wave energy angular spreading function. The wave energy spectrum was discretised into 19 directional ranges Δμi and 103 frequency ranges Δωi per direction. To generate independent realisations of irregular seaway, for each realisation the phases εi were randomly selected in the interval [0,2π), frequencies ωi and directions μi were randomly selected within their ranges Δωi and Δμi, respectively, and the amplitudes ai were assumed equal for all components. To generate random values, pseudo-random number generator with the internal computer timer as the seed number was used.

3.5.3 To ensure that stability failures in numerical simulations satisfy the assumptions of a Poisson process, self-repetition effects were avoided by limiting the maximum simulation time in each seaway realisation by 1 hour, transient hydrodynamic effects at the beginning of each simulation were eliminated by switching off the counter of stability failures and simulation timer during the initial 50 roll periods, and auto-correlation of big roll motions was avoided by stopping a simulation after the first stability failure.

3.5.4 Assessment was performed for parametric roll stability failure mode, using as the criterion the maximum (over all required design situations) of the upper boundary of the 95%-confidence interval of the short-term stability failure rate. For acceptance, this criterion should not exceed the threshold equal to one stability failure in 2 hours in design sea states with probability density 10‑5 (m⋅s)‑1. The design situations for parametric roll stability failure mode correspond to zero forward speed in head and following mean wave directions. The mean zero-upcrossing wave periods from 3.5 s to 17.5 s with the step of 1.0 s were used. For each wave period, significant wave height was selected according to Table 3.5.3.3.4 of the Guidelines.

3.5.5 The examples concern a 1700 TEU container ship in loading conditions with *GM*=1.7 m, 1.8 m, …, 2.2 m.

3.5.6 Each simulation was run for 1 hour but stopped after the first stability failure or when the sufficient simulation time for acceptance  was achieved, with. If the sufficient simulation time for acceptance  was achieved during a simulation, no further simulations in the considered design situation were conducted and the loading condition was considered as *acceptable* in the considered design situation; otherwise the following parameters were defined after a simulation: realisation number i, the number of failures ΔN (1 or 0) in the realisation and duration of simulation Δt (equal to time to failure if realisation ended with a stability failure or 1 hour if stability failure was not encountered).

3.5.7 From these parameters, the following variables were estimated: the total number of failures N and the total simulation time tt (increased by ΔN and Δt, respectively, after each realisation), the sample mean time to failure  and the maximum likelihood estimate of the failure rate . After that the *not-acceptance* condition was checked: if , where  and , no further simulations were conducted for the considered loading condition, which was judged as *not acceptable*.

3.5.8 The functions  and  were calculated with MS Excel as chisq.inv.rt(α/2;2\*N) and chisq.inv(α/2;2\*N), respectively.

3.5.9 Tables 3.5.1 to 3.5.4 show results for loading conditions with *GM*=1.8 m (not acceptable) and 1.9 m (acceptable) together with the required simulation time ts until each stability failure and cumulative simulation time over all wave periods tc (for brevity, the results are shown not after each simulation but only after stability failures).

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table 3.5.1 Assessment results for GM=1.8 m in head waves (A and F denote *acceptance* and *not acceptance* decisions, respectively) | | | | | | | | | | |
| Tz,s | N | Ti,s | ,s | tA,s | A | ,s | F | ts,s | tc,s | tc,hour |
| 7.5 | 1 | 9.57e+2 | 9.57e+2 | 2.66e+4 | - | 1.82e+2 | - | 9.57e+2 | 9.57e+2 | 0.3 |
|  | 2 | 1.01e+4 | 5.54e+3 | 3.92e+4 | - | 8.72e+2 | - | 1.01e+4 | 1.11e+4 | 3.1 |
|  | 3 | 5.02e+2 | 3.86e+3 | 4.09e+4 | - | 1.48e+3 | - | 5.02e+2 | 1.16e+4 | 3.2 |
|  | 4 | 7.49e+2 | 3.08e+3 | 5.15e+4 | - | 1.96e+3 | - | 7.49e+2 | 1.23e+4 | 3.4 |
|  | 5 | 4.94e+3 | 3.46e+3 | 6.14e+4 | - | 2.34e+3 | - | 4.94e+3 | 1.73e+4 | 4.8 |
|  | 6 | 9.50e+2 | 3.04e+3 | 6.67e+4 | - | 2.64e+3 | - | 9.50e+2 | 1.82e+4 | 5.1 |
|  | 7 | 1.03e+4 | 4.08e+3 | 7.58e+4 | - | 2.89e+3 | - | 1.03e+4 | 2.85e+4 | 7.9 |
|  | 8 | 4.02e+2 | 3.62e+3 | 7.53e+4 | - | 3.11e+3 | - | 4.02e+2 | 2.89e+4 | 8.0 |
|  | 9 | 1.28e+3 | 3.36e+3 | 8.46e+4 | - | 3.29e+3 | - | 1.28e+3 | 3.02e+4 | 8.4 |
|  | 10 | 7.48e+3 | 3.77e+3 | 9.28e+4 | - | 3.45e+3 | - | 7.48e+3 | 3.77e+4 | 10.5 |
|  | 11 | 1.29e+3 | 3.54e+3 | 9.47e+4 | - | 3.59e+3 | F | 1.29e+3 | 3.90e+4 | 10.8 |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table 3.5.2 Assessment results for GM=1.8 m in following waves (A and F denote *acceptance* and *not acceptance* decisions, respectively) | | | | | | | | | | |
| Tz,s | N | Ti,s | ,s | tA,s | A | ,s | F | ts,s | tc,s | tc,hour |
| 7.5 | 1 | 2.46e+3 | 2.46e+3 | 2.66e+4 | - | 1.82e+2 | - | 2.46e+3 | 2.46e+3 | 0.7 |
|  | 2 | 2.65e+3 | 2.56e+3 | 3.77e+4 | - | 8.72e+2 | - | 2.65e+3 | 5.11e+3 | 1.4 |
|  | 3 | 3.58e+3 | 2.90e+3 | 4.69e+4 | - | 1.48e+3 | - | 3.58e+3 | 8.70e+3 | 2.4 |
|  | 4 | 7.58e+2 | 2.36e+3 | 5.44e+4 | - | 1.96e+3 | - | 7.58e+2 | 9.45e+3 | 2.6 |
|  | 5 | 1.44e+3 | 2.18e+3 | 6.43e+4 | - | 2.34e+3 | F | 1.44e+3 | 1.09e+4 | 3.0 |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table 3.5.3 Assessment results for GM=1.9 m in head waves (A and F denote *acceptance* and *not acceptance* decisions, respectively) | | | | | | | | | | |
| Tz,s | N | Ti,s | ,s | tA,s | A | ,s | F | ts,s | tc,s | tc,hour |
| 7.5 | 1 | 2.11e+2 | 2.11e+2 | 2.66e+4 | - | 1.82e+2 | - | 2.11e+2 | 2.11e+2 | 0.1 |
|  | 2 | 3.15e+4 | 1.59e+4 | 3.99e+4 | - | 8.72e+2 | - | 3.15e+4 | 3.17e+4 | 8.8 |
|  | 3 | 4.31e+4 | 2.49e+4 | 2.03e+4 | A | 1.48e+3 | - | 2.03e+4 | 5.20e+4 | 14.4 |
| 8.5 | 1 | 4.76e+4 | 4.76e+4 | 2.66e+4 | A | 1.82e+2 | - | 2.66e+4 | 7.86e+4 | 21.8 |
| 9.5 | 1 | 3.01e+4 | 3.01e+4 | 2.66e+4 | A | 1.82e+2 | - | 2.66e+4 | 1.05e+5 | 29.2 |
| 10.5 | 1 | 2.45e+4 | 2.45e+4 | 2.66e+4 | - | 1.82e+2 | - | 2.45e+4 | 1.30e+5 | 36.0 |
|  | 2 | 2.04e+4 | 2.25e+4 | 1.56e+4 | A | 8.72e+2 | - | 1.56e+4 | 1.45e+5 | 40.3 |
| 11.5 | 1 | 1.56e+4 | 1.56e+4 | 2.66e+4 | - | 1.82e+2 | - | 1.56e+4 | 1.61e+5 | 44.7 |
|  | 2 | 3.49e+4 | 2.53e+4 | 2.45e+4 | A | 8.72e+2 | - | 2.45e+4 | 1.85e+5 | 51.5 |
| 12.5 | 1 | 1.57e+4 | 1.57e+4 | 2.66e+4 | - | 1.82e+2 | - | 1.57e+4 | 2.01e+5 | 55.9 |
|  | 2 | 1.99e+4 | 1.78e+4 | 2.44e+4 | - | 8.72e+2 | - | 1.99e+4 | 2.21e+5 | 61.4 |
|  | 3 | 6.82e+4 | 3.46e+4 | 1.64e+4 | A | 1.48e+3 | - | 1.64e+4 | 2.37e+5 | 65.9 |
| 13.5 | 1 | 6.05e+3 | 6.05e+3 | 2.66e+4 | - | 1.82e+2 | - | 6.05e+3 | 2.43e+5 | 67.6 |
|  | 2 | 8.33e+4 | 4.47e+4 | 3.41e+4 | A | 8.72e+2 | - | 3.41e+4 | 2.78e+5 | 77.1 |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table 3.5.4 Assessment results for GM=1.9 m in following waves (A and F denote *acceptance* and *not acceptance* decisions, respectively) | | | | | | | | | | |
| Tz,s | N | Ti,s | ,s | tA,s | A | ,s | F | ts,s | tc,s | tc,hour |
| 7.5 | 1 | 2.39e+3 | 2.39e+3 | 2.66e+4 | - | 1.82e+2 | - | 2.39e+3 | 2.39e+3 | 0.7 |
|  | 2 | 1.57e+5 | 7.96e+4 | 3.77e+4 | A | 8.72e+2 | - | 3.77e+4 | 4.01e+4 | 11.1 |
| 8.5 | 1 | 5.15e+3 | 5.15e+3 | 2.66e+4 | - | 1.82e+2 | - | 5.15e+3 | 4.53e+4 | 12.6 |
|  | 2 | 1.08e+3 | 3.11e+3 | 3.50e+4 | - | 8.72e+2 | - | 1.08e+3 | 4.63e+4 | 12.9 |
|  | 3 | 2.78e+4 | 1.13e+4 | 4.58e+4 | - | 1.48e+3 | - | 2.78e+4 | 7.41e+4 | 20.6 |
|  | 4 | 6.56e+3 | 1.01e+4 | 2.91e+4 | - | 1.96e+3 | - | 6.56e+3 | 8.07e+4 | 22.4 |
|  | 5 | 8.82e+4 | 2.58e+4 | 3.32e+4 | A | 2.34e+3 | - | 3.32e+4 | 1.14e+5 | 31.6 |
| 9.5 | 1 | 1.89e+4 | 1.89e+4 | 2.66e+4 | - | 1.82e+2 | - | 1.89e+4 | 1.33e+5 | 36.9 |
|  | 2 | 1.09e+3 | 9.98e+3 | 2.12e+4 | - | 8.72e+2 | - | 1.09e+3 | 1.34e+5 | 37.2 |
|  | 3 | 2.78e+3 | 7.58e+3 | 3.21e+4 | - | 1.48e+3 | - | 2.78e+3 | 1.37e+5 | 37.9 |
|  | 4 | 1.16e+4 | 8.59e+3 | 4.04e+4 | - | 1.96e+3 | - | 1.16e+4 | 1.48e+5 | 41.2 |
|  | 5 | 2.11e+4 | 1.11e+4 | 3.94e+4 | - | 2.34e+3 | - | 2.11e+4 | 1.69e+5 | 47.0 |
|  | 6 | 8.16e+3 | 1.06e+4 | 2.85e+4 | - | 2.64e+3 | - | 8.16e+3 | 1.78e+5 | 49.3 |
|  | 7 | 2.73e+3 | 9.49e+3 | 3.03e+4 | - | 2.89e+3 | - | 2.73e+3 | 1.80e+5 | 50.1 |
|  | 8 | 1.48e+4 | 1.02e+4 | 3.74e+4 | - | 3.11e+3 | - | 1.48e+4 | 1.95e+5 | 54.2 |
|  | 9 | 2.48e+4 | 1.18e+4 | 3.23e+4 | - | 3.29e+3 | - | 2.48e+4 | 2.20e+5 | 61.1 |
|  | 10 | 8.19e+3 | 1.14e+4 | 1.70e+4 | - | 3.45e+3 | - | 8.19e+3 | 2.28e+5 | 63.4 |
|  | 11 | 9.81e+3 | 1.13e+4 | 1.82e+4 | - | 3.59e+3 | - | 9.81e+3 | 2.38e+5 | 66.1 |
|  | 12 | 5.55e+4 | 1.50e+4 | 1.77e+4 | A | 3.72e+3 | - | 1.77e+4 | 2.56e+5 | 71.0 |
| 10.5 | 1 | 6.09e+4 | 6.09e+4 | 2.66e+4 | A | 1.82e+2 | - | 2.66e+4 | 2.82e+5 | 78.4 |
| 11.5 | 1 | 2.91e+3 | 2.91e+3 | 2.66e+4 | - | 1.82e+2 | - | 2.91e+3 | 2.85e+5 | 79.2 |
|  | 2 | 2.01e+4 | 1.15e+4 | 3.72e+4 | - | 8.72e+2 | - | 2.01e+4 | 3.05e+5 | 84.8 |
|  | 3 | 1.97e+3 | 8.34e+3 | 2.90e+4 | - | 1.48e+3 | - | 1.97e+3 | 3.07e+5 | 85.3 |
|  | 4 | 1.18e+4 | 9.21e+3 | 3.81e+4 | - | 1.96e+3 | - | 1.18e+4 | 3.19e+5 | 88.6 |
|  | 5 | 2.78e+4 | 1.29e+4 | 3.69e+4 | - | 2.34e+3 | - | 2.78e+4 | 3.47e+5 | 96.3 |
|  | 6 | 4.75e+3 | 1.16e+4 | 1.94e+4 | - | 2.64e+3 | - | 4.75e+3 | 3.51e+5 | 97.6 |
|  | 7 | 4.78e+3 | 1.06e+4 | 2.47e+4 | - | 2.89e+3 | - | 4.78e+3 | 3.56e+5 | 99.0 |
|  | 8 | 1.57e+3 | 9.46e+3 | 2.97e+4 | - | 3.11e+3 | - | 1.57e+3 | 3.58e+5 | 99.4 |
|  | 9 | 2.96e+4 | 1.17e+4 | 3.78e+4 | - | 3.29e+3 | - | 2.96e+4 | 3.87e+5 | 107.6 |
|  | 10 | 1.15e+4 | 1.17e+4 | 1.77e+4 | - | 3.45e+3 | - | 1.15e+4 | 3.99e+5 | 110.8 |
|  | 11 | 1.07e+3 | 1.07e+4 | 1.57e+4 | - | 3.59e+3 | - | 1.07e+3 | 4.00e+5 | 111.1 |
|  | 12 | 1.81e+4 | 1.13e+4 | 2.39e+4 | - | 3.72e+3 | - | 1.81e+4 | 4.18e+5 | 116.1 |
|  | 13 | 2.14e+2 | 1.05e+4 | 1.50e+4 | - | 3.83e+3 | - | 2.14e+2 | 4.18e+5 | 116.2 |
|  | 14 | 8.84e+3 | 1.04e+4 | 2.39e+4 | - | 3.94e+3 | - | 8.84e+3 | 4.27e+5 | 118.6 |
|  | 15 | 1.97e+4 | 1.10e+4 | 2.41e+4 | - | 4.03e+3 | - | 1.97e+4 | 4.47e+5 | 124.1 |
|  | 16 | 2.24e+4 | 1.17e+4 | 1.34e+4 | A | 4.12e+3 | - | 1.34e+4 | 4.60e+5 | 127.8 |
| 12.5 | 1 | 5.91e+3 | 5.91e+3 | 2.66e+4 | - | 1.82e+2 | - | 5.91e+3 | 4.66e+5 | 129.5 |
|  | 2 | 1.42e+4 | 1.00e+4 | 3.42e+4 | - | 8.72e+2 | - | 1.42e+4 | 4.80e+5 | 133.4 |
|  | 3 | 1.14e+4 | 1.05e+4 | 3.19e+4 | - | 1.48e+3 | - | 1.14e+4 | 4.92e+5 | 136.6 |
|  | 4 | 4.34e+3 | 8.96e+3 | 3.16e+4 | - | 1.96e+3 | - | 4.34e+3 | 4.96e+5 | 137.8 |
|  | 5 | 8.40e+3 | 8.85e+3 | 3.79e+4 | - | 2.34e+3 | - | 8.40e+3 | 5.05e+5 | 140.1 |
|  | 6 | 4.77e+3 | 8.17e+3 | 3.98e+4 | - | 2.64e+3 | - | 4.77e+3 | 5.09e+5 | 141.5 |
|  | 7 | 9.33e+3 | 8.34e+3 | 4.50e+4 | - | 2.89e+3 | - | 9.33e+3 | 5.19e+5 | 144.1 |
|  | 8 | 4.89e+3 | 7.91e+3 | 4.55e+4 | - | 3.11e+3 | - | 4.89e+3 | 5.24e+5 | 145.4 |
|  | 9 | 1.03e+4 | 8.17e+3 | 5.02e+4 | - | 3.29e+3 | - | 1.03e+4 | 5.34e+5 | 148.3 |
|  | 10 | 1.85e+3 | 7.54e+3 | 4.95e+4 | - | 3.45e+3 | - | 1.85e+3 | 5.36e+5 | 148.8 |
|  | 11 | 1.44e+4 | 8.16e+3 | 5.70e+4 | - | 3.59e+3 | - | 1.44e+4 | 5.50e+5 | 152.8 |
|  | 12 | 9.41e+3 | 8.27e+3 | 5.19e+4 | - | 3.72e+3 | - | 9.41e+3 | 5.59e+5 | 155.4 |
|  | 13 | 8.08e+2 | 7.69e+3 | 5.17e+4 | - | 3.83e+3 | - | 8.08e+2 | 5.60e+5 | 155.6 |
|  | 14 | 2.15e+4 | 8.68e+3 | 6.00e+4 | - | 3.94e+3 | - | 2.15e+4 | 5.82e+5 | 161.6 |
|  | 15 | 1.18e+4 | 8.89e+3 | 4.76e+4 | - | 4.03e+3 | - | 1.18e+4 | 5.94e+5 | 164.9 |
|  | 16 | 3.54e+3 | 8.56e+3 | 4.48e+4 | - | 4.12e+3 | - | 3.54e+3 | 5.97e+5 | 165.9 |
|  | 17 | 1.49e+4 | 8.93e+3 | 5.02e+4 | - | 4.19e+3 | - | 1.49e+4 | 6.12e+5 | 170.0 |
|  | 18 | 2.81e+4 | 9.99e+3 | 4.42e+4 | - | 4.27e+3 | - | 2.81e+4 | 6.40e+5 | 177.8 |
|  | 19 | 1.23e+4 | 1.01e+4 | 2.50e+4 | - | 4.33e+3 | - | 1.23e+4 | 6.52e+5 | 181.2 |
|  | 20 | 1.68e+4 | 1.04e+4 | 2.15e+4 | - | 4.40e+3 | - | 1.68e+4 | 6.69e+5 | 185.9 |
|  | 21 | 7.11e+2 | 9.98e+3 | 1.35e+4 | - | 4.46e+3 | - | 7.11e+2 | 6.70e+5 | 186.1 |
|  | 22 | 6.27e+3 | 9.81e+3 | 2.15e+4 | - | 4.51e+3 | - | 6.27e+3 | 6.76e+5 | 187.8 |
|  | 23 | 3.55e+4 | 1.09e+4 | 2.40e+4 | A | 4.56e+3 | - | 2.40e+4 | 7.00e+5 | 194.5 |
| 13.5 | 1 | 1.18e+3 | 1.18e+3 | 2.66e+4 | - | 1.82e+2 | - | 1.18e+3 | 7.01e+5 | 194.8 |
|  | 2 | 7.55e+4 | 3.83e+4 | 3.89e+4 | A | 8.72e+2 | - | 3.89e+4 | 7.40e+5 | 205.6 |
| 14.5 | 1 | 1.34e+6 | 1.34e+6 | 2.66e+4 | A | 1.82e+2 | - | 2.66e+4 | 7.67e+5 | 213.0 |

3.5.10 *Not-acceptance* requires much less simulation or testing time (since not satisfaction of the threshold in only one situation is sufficient): for GM=1.7 m, *not acceptance* requires 87 min. if tests in head waves are done first and 2.7 min. if tests in following waves are done first; for GM=1.8 m, *not acceptance* requires 10.8 hours or 3.0 hours, respectively. For *acceptance*, significantly more simulation or testing time is required since all design situations must be considered and the failure rate is lower: for GM=1.9 m, *acceptance* requires 290.1 hours of simulation time (from which 77.1 hour in head waves and 213.0 hours in following); for GM=2.0 m, 262.8 hours are required (69.3 hours in head waves and 193.5 hours in following). Thus, following wave situations are more efficient for *not acceptable* loading conditions while head waves are more efficient for *acceptable* loading conditions.

3.5.11 The final decision and the total required simulation or testing time to make this decision are summarised in Table 3.5.5, separately for assessment in head waves, in following waves and in both head and following waves.

|  |  |  |  |
| --- | --- | --- | --- |
| Table 3.5.5 Final decision and required simulation or testing time if assessment is done in head, in following or both in head and following waves (results are shown as “result for head waves + result for following waves = result for both wave directions”) | | | |
| GM, m | Decision (A=acceptance, F=not acceptance) | Total simulation time, hours | Total simulation time, % |
|  | | | |
| 1.7 | F + F = F | 1.4 + **0.1** = 1.5 | 97 + 3 = 100 |
| 1.8 | F + F = F | 10.8 + **3.1** = 13.9 | 78 + 22 = 100 |
| 1.9 | A + A = A | **77.1** + 213.0 = 290.1 | 27 + 73 = 100 |
| 2.0 | A + A = A | **69.3** + 193.5 = 262.8 | 36 + 74 = 100 |
|  | | | |
| total | | **158.6** + 409.6 = 568.2 | 28 + 72 = 100 |

12 Much less simulation or testing time (bold figures in Table 3.5.5) is required for *not acceptable* loading conditions in following waves and for *acceptable* loading conditions in head waves. Since assessment for *acceptable* loading conditions requires much more time than for *not acceptable* loading conditions, on the average the assessment in head waves requires significantly less time than in following waves.

**3.6 Background information for direct counting**

**3.6.1 Introduction**

3.6.1.1 Background information for the direct counting procedure and for the direct stability assessment in design sea states is provided here.

3.6.1.2 *Direct counting* is counting of the number of stability failures per given exposure time. Direct counting is used in the probabilistic design stability assessment in design situations, as well as (in combination with statistical extrapolation, which itself uses direct counting) in the full probabilistic direct stability assessment and in operational measures.

3.6.1.3 Counting of the number of stability failures per given exposure time assumes a stationary Poisson process. For the real operation, the stationarity assumption is justified since both design assessment and operational measures consider ensemble statistics over a big number of ships, each of which operates in stationary conditions for unlimited time. Poisson process assumption requires, in addition, that stability failures happen independently, i.e. occurrence of one failure does not affect the probability of occurrence of a second failure. The validity of this assumption is based on two heuristic considerations: *clumping heuristic* (although big roll motions tend to appear in groups, occurrence of such groups may be independent), and *rarity heuristic* (rare events tend to be independent).

3.6.1.4 To ensure that numerical simulations or model tests satisfy the requirements of a stationary Poisson process, special procedures are required that are considered below.

**3.6.2 Definition and characteristics of Poisson process**

3.6.2.1 First, define a *counting process* as a stochastic process N(t), where the integer random variable N counts the total number of stability failures (the usual term in mathematics is *arrivals*) in the time interval from 0 up to and including time t. The number of failures in time interval (s,t], equal to N(t)‑N(s), is called *increment*. We consider such counting processes in which the increments are *independent* (i.e. numbers of failures in non-overlapping time intervals are independent) and *stationary* (i.e. the number of failures depends only on the length of a time interval and does not depend on its location in time).

3.6.2.2 There are several equivalent definitions of a Poisson process; here, a Poisson process with a constant rate r>0 is defined as a counting process N(t) which has independent increments (note that this definition automatically ensures stationary increments) and where the number of arrivals N(t) in any time interval of length t satisfies the Poisson distribution with the mean rt, i.e.

|  |  |
| --- | --- |
| p{N(t)=k}=(rt)k⋅e−rt/k! for k=0, 1, ... | (3.6.1) |

3.6.2.3 The probability density function of the Poisson distribution, f(k)=p{N(t)=k}, given by eq. (3.6.1), is the probability that the number of arrivals during a time interval t is equal to k.

3.6.2.4 Properties of a Poisson process:

.1 *superposition property*: sum of independent Poisson processes N1, …, Nk, i.e. N1+∙∙∙+Nk, is a Poisson process with the rate r1+∙∙∙+rk. Therefore, failure rates can be found separately for each stability failure mode, e.g. using different numerical methods, and summed to give total stability failure rate;

.2 conversely, if the sum of two independent random variables is Poisson distributed, so are each of these two variables;

.3 *random split property*: if each arrival of a Poisson process N(t) with rate r is randomly tagged as either process N1(t), with probability p, or N2(t), with probability 1‑p, then the two resulting processes N1(t) and N2(t) are independent Poisson processes with rates rp and r(1‑p), respectively; and

.4 *thinning property*: if each arrival of a Poisson process with rate r is randomly marked, with probability p, then the marked process is a Poisson process with rate rp.

.5 *mean* of a Poisson process (the mean number of arrivals per interval *t*) is

|  |  |
| --- | --- |
|  | (3.6.2) |

Thus, the rate r is equal to the expected number of arrivals per time unit.

.6 The variance of the Poisson process is equal to the mean, Var{N(t)}=rt.

3.6.2.5 A special case of eq. (3.6.1) is *k*=0, which gives the probability that no stability failures occur from time 0 to time *t*:

|  |  |
| --- | --- |
| p≡p{N(t)=0}=e−rt | (3.6.3) |

3.6.2.6 From eq. (3.6.3), find the probability that at least one stability failure happens during time interval *t*, i.e. that *k*>0 (loosely formulated: “probability of stability failure during time *t*”) as

|  |  |
| --- | --- |
| p\*≡p{N(t)>0}=1−p{N(t)=0}=1−p=1−e−rt | (3.6.4) |

3.6.2.7 A Poisson process can be seen as a sequence of time intervals T1 (from t=0 to the first failure), T2 (between the first and second failures) etc., which are random variables. Nothe that the probability that the time until the first failure exceeds t, i.e. p{T1>t}, is the same as the probability that no failures occur before time t, i.e. p{N(t)=0}, which is equal to e−rt, eq. (3.6.3). Thus, p{T1>t}=e−rt which means that T1 is exponentially distributed (similarly it can be shown that all time intervals Ti are exponentially distributed random variables with the same rate r). Therefore, Poisson process can also be defined as a counting process N(t) in which time intervals between arrivals are independent random variables, exponentially distributed with rate r (note that this definition automatically ensures independent and stationary increments):

|  |  |
| --- | --- |
| p{T>t}=e−rt for t>0 and 0 otherwise | (3.6.5) |

3.6.2.8 The most important property of the exponential distribution is its *memoryless property*: if failure has not occurred until time t, the distribution of the remaining waiting time is the same as the distribution of the original waiting time, i.e. remaining waiting time has no memory of previous waiting. The exponential distribution is the only continuous distribution with this property: if the time intervals between arrivals are not exponential, the process will not be a Poisson process since it does not have stationary and independent increments.

3.6.2.9 Other properties of exponential distribution:

.1 correspondingly to sum of Poisson processes, if T1,…,Tk are independent exponentially distributed random variables with rates r1,…,rk, then min(T1,…,Tk) is exponentially distributed with rate r1+∙∙∙+rk, and the index of the variable that achieves the minimum is distributed according to the law p{i | Ti=min(T1,…,Tk)}=ri/(r1+∙∙∙rk);

.2 according to eq. (3.6.4), the cumulative density function of time to failure is

|  |  |
| --- | --- |
| F(t) ≡p{0<T<t}=1−p{T>t}=1−e−rt for t>0 and 0 otherwise | (3.6.6) |

.3 probability density function of exponential distribution, i.e. of the time intervals between arrivals in a Poisson process, is

|  |  |
| --- | --- |
| f(t)=dF(t)/dt=re−rt for t>0 and 0 otherwise | (3.6.7) |

.4 mean of exponentially distributed random variable *T* (i.e. mean time between stability failures) is

|  |  |
| --- | --- |
|  | (3.6.8) |

.5 the second moment , thus the variance of the time between failures is , and the standard deviation of time between failures is equal to

|  |  |
| --- | --- |
| σ{T}=(Var{T})1/2=1/r= | (3.6.9) |

3.6.2.10 Properties in paragraph 3.6.2.9.5 mean that any statistical characteristic of the exponential distribution, including variance and standard deviation, is known once the stability failure rate is known. To confirm eq. (3.6.9), Fig. 1 shows the ratio  depending on the number of counted failures N, and Fig. 2 shows the estimate of the standard deviation σ{T} as a function of the estimate of the mean time to failure  after N=200 counted failures.

|  |  |
| --- | --- |
|  |  |
|  |  |
| Fig. 3.6.1 Ratio of estimate of standard deviation of time to failure to estimate of mean time to failure vs. number of failures. | Fig. 3.6.2 Estimate of standard deviation of time to failure vs. estimate of mean time to failure from 200 simulated failures. |

**3.6.3. Definition of failure rate from sample data**

3.6.3.1 Both the Poisson distribution and the corresponding exponential distribution are defined by a single parameter, stability failure rate r. To define it from a series of numerical simulations or model tests, the time intervals Ti between failures should be measured and the *sample mean time to failure* is calculated after N failures as

|  |  |
| --- | --- |
|  | (3.6.10) |

3.6.3.2 To define stability failure rate,

.1 first define the joint probability density function Lr of all individual time intervals Ti: since Ti are independent, eq. (3.6.7) leads to

|  |  |
| --- | --- |
|  | (3.6.11) |

.2 the most probable value of r is the value that maximises Lr. More convenient is to maximise , which is possible since logarithm is a monotonously increasing function, thus Lr and ln(Lr) have maximum at the same r. Maximum of ln(Lr) can be defined from the condition dln(Lr)/dr=0, i.e.

|  |  |
| --- | --- |
|  | (3.6.12) |

.3 From eq. (3.6.12), the *maximum likelihood estimate of the stability failure rate* is

|  |  |
| --- | --- |
|  | (3.6.13) |

.4 Introduce the total simulation (or model testing) time tt as

|  |  |
| --- | --- |
|  | (3.6.14) |

then eq. (3.6.13) can be written as

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|  | (3.6.15) |

3.6.3.3 A (1−*α*)⋅100%-confidence interval for the stability failure rate, where *α* is a small value (e.g. for a 95%-confidence interval, *α*=1−95/100=0.05), can be estimated from

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| --- | --- |
|  | (3.6.16) |

3.6.3.4 In eq. (3.6.16),  is the quantile function (corresponding to a lower tail area, equal to the cumulative probability *p*) of the χ2-distribution with *f* degrees of freedom. The function  is available in many software packages; e.g. in MS Excel, it can be calculated as chisq.inv(p;f) or, alternatively, as chisq.inv.rt(1-p;f). Correspondingly, the upper-tail quantile function  can be calculated as chisq.inv.rt(p;f) or chisq.inv(1-p;f), see Fig. 3.6.3.

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| --- | --- |
|  | Fig. 3.6.3 Examples of functions  and , calculated with MS Excel as chisq.inv(p;f) and chisq.inv.rt(p;f), respectively, for *f* = 1, 5, 10 and 50 degrees of freedom vs. cumulative probability *p* |

3.6.3.5 Resolving eq. (3.6.16) with respect to r gives , thus the upper rU and lower rL boundaries of a (1−*α*)⋅100%-confidence interval for the stability failure rate can be calculated as

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| --- | --- |
|  | (3.6.17) |
|  | (3.6.18) |

3.6.3.6 Using these estimates, any other characteristic of the process can be calculated with formulae from section 3.6.2. For example, the upper boundary of the (1−*α*)⋅100%-confidence interval of probability of stability failure during any time interval t is

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| --- | --- |
|  | (3.6.19) |

3.6.3.7 Note that alternatively, the probability that at least one failure happens per given exposure time t can be defined directly from results of M simulations of constant duration t, as p≈N/M, where N is the total number of failures observed in M simulations:

.1 substituting p=N/M in eq. (3.6.1) leads to the following estimate for failure rate:

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|  | (3.6.20) |

.2 however, using this alternative procedure does not provide any advantages regarding the total simulation time or procedure simplicity;

.3 moreover, estimate (3.6.20) is not the most likelihood estimate of failure rate but an approximate result, which improves in accuracy and converges to the result of the proposed direct counting method when M increases to infinity while the total simulation time tt is kept constant (i.e. when t→0).

**3.6.4. Long-term statistics**

3.6.4.1 Consider the long-term operation as a sum of a number of stationary Poisson processes, each of which has a constant rate ri and happens in a stationary sailing situation which is encountered with a probability pi. Then, applying the *sum property* and *tagging property* of a stationary Poisson process, the long-term operation can be considered a stationary Poisson process with a constant rate , which is equal to

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|  | (3.6.21) |

3.6.4.2 All properties of Poisson process remain applicable by using the mean rate , e.g.

.1 distribution of the number of failures in a time interval from t to t+τ, i.e. the probability that k failures happen in time interval from t to t+τ, is equal to

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|  | (3.6.22) |

.2 probability that no failures occur from time t to time t+τ is

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|  | (3.6.23) |

.3 probability that at least one failure happens from time t to time t+τ (“probability of failure during time τ”) is

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|  | (3.6.24) |

3.6.4.3 Therefore, the problem reduces to the definition of a constant rate ri in each stationary sailing situation from either numerical simulations or model tests in such a way that ensures that the process is a stationary Poisson process.

**3.6.5. Cautions in numerical simulations or model tests**

3.6.5.1 Numerical simulations (or model tests) and the used counting procedure should ensure stationarity of the process and independence of the counted stability failures. Using the ergodicity property, the following straightforward procedure seems suitable: run one, long enough, simulation of roll motion and, after each encountered stability failure, increase the number N of failures by 1, increase total simulation time tt by the simulation time from the previous failure, update the estimate of failure rate , eq. (3.6.15), and the estimate of the upper boundary of its (1−*α*)⋅100%-confidence interval , eq. (3.6.17), and compare rU with the standard: once rU is less than the acceptance standard, the simulation can be stopped and the loading condition can be considered acceptable.

3.6.5.2. However, collecting sufficient statistics in one sufficiently long run may be impossible in practice since the duration of numerical simulations or model tests is limited:

.1 the duration of each model test is limited by wave reflection effects;

.2 the duration of numerical simulations is limited due to self-repetition effects, which violate the requirement of the independence of failures, if the sea state is modelled as a finite sum of harmonic components:

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| --- | --- |
|  | (3.6.25) |

where ai={2Sζζ(ωi)D(μi)ΔωiΔμi}1/2 are amplitudes, ωi frequencies, μi directions and εi phases of harmonic components, randomly selected in the interval [0,2*π*), Sζζ is the wave energy spectrum and D is the wave energy angular spreading function;

.3 note that for resonance phenomena, such as parametric and synchronous roll, perfect repetition is not required – it is bad enough to have a partial repetition in a relevant narrow band of encounter frequencies.

3.6.5.3 A possible solution is to generate multiple independent *realisations* of the same sea state, randomly varying phases  for each realisation, and simulate ship motions in each such realisation for a limited time. It is recommended to randomly vary also frequencies *ωi* and directions *μi* of components, and perhaps also amplitudes *ai*, using normal distributions with specified standard deviations. To generate random values, pseudo-random number generators can be used, which apply a user-specified integer as a seed number.

3.6.5.4 For a set of simulations in multiple realisations of the same sea state, eq. (3.6.15) is still applicable; in this equation, N and tt should be taken as the total number of stability failures and the total simulation time, respectively, over all realisations.

3.6.5.5 Another problem are transient hydrodynamic effects occurring at the beginning of each simulation, which violate stationarity requirement. To address this, certain time after the start of each simulation (as a recommendation, 50 roll periods) should be excluded from post-processing, i.e. this simulation time should not be included in tt while failures during this time should not be counted in N in eq. (3.6.15).

3.6.5.6 Finally, the independence of stability failures in numerical simulations may be violated by the autocorrelation of big roll motions since big roll amplitudes caused by resonance tend to appear in groups. Note that neglecting this effect may lead to an over-estimation of the sample mean stability failure rate but also under-estimation of the size of the confidence interval. To neutralise this effect, the following solutions are possible:

.1 to run each simulation only up to the first encountered failure; and

.2 to switch off the simulation timer tt and the stability failure counter N after an encountered failure until the envelope of the autocorrelation function of roll motion reduces to a specified level;

.3 the two options 3.6.5.6.1 and 3.6.5.6.2 lead to similar unproductive losses: in the former method, due to transient effects at the start of a new simulation, in the latter, for the decay of the autocorrelation function of roll motion. However, the former method is simpler. Besides, another benefit of the former method is that restarting also takes care of repeatability effects: for the relevant durations of simulations and relevant stability failure rates, a stability failure is encountered not in each simulation, therefore, after the first encountered failure, there is no sense to wait for the second in the same simulation.

3.6.5.7 After removing portions of time histories of roll motion, affected by repeatability effects, initial transients and auto-correlation of stability failures, remaining pieces represent a single stationary Poisson process: removed pieces can be disregarded due to the memoryless property, durations of the remaining pieces may be arbitrary (in particular, equal), and it does not matter whether a failure was encountered in each simulation or not, Fig. 3.6.4. This means that eq. (3.6.15) can be used for the most likelihood estimate of the failure rate, with N and tt representing sums over all remaining pieces of simulations. Similarly, the sample mean time to failure is , and all other formulae from sections 2 and 3 can be directly applied.

3.6.5.8 To investigate repeatability effects, parametric resonance in head waves and synchronous resonance in beam waves were simulated for a systematically varied significant wave height in two types of simulations: in one, denoted ‘limited’ for brevity, the simulation time was limited to 3 hours (while simulations were stopped in any case after the first failure); and in the other (‘unlimited’), simulations were run always until the first failure. Thus, after each ‘limited’ simulation, N was increased by 1 while tt was increased by the time to failure in case of failure, and N was not changed while tt was increased by 3 hours if a simulation ended in 3 hours without failure. After each ‘unlimited’ simulation, N was increased by 1 and tt was increased by the time to failure.

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|  | Fig. 3.6.4 Roll motion in multiple realisations of sea state (top) and resulting Poisson process (bottom) |

3.6.5.9 To verify the absence of self-repetition effects, quantile diagrams (QQ diagrams) can be used. Fig. 3.6.5 shows these diagrams derived from the ‘limited’ and ‘unlimited’ simulations. Since the cumulative density function of an exponentially distributed time to failure is F(t) = 1 − e−rt for t > 0, the ratio  should be equal to −ln(1 − Fi) (blue dashed lines in Fig. 3.6.5). The cumulative density function Fi was calculated from the sample data as i / (N + 1), where i is the index of a stability failure when stability failures are sorted in ascending order of Ti. Fig. 3.6.5 shows that the ‘unlimited’ simulations over-estimate failure time compared to the exponential distribution and ‘limited’ simulations for cases with large time to failure. This is due to self-repetition effects: the same ‘uncritical’ realisation, i.e. realisation where a stability failure does not happen soon, repeats itself again and again (since repetition is not exact, failure may eventually happen but much later than it should).

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| Fig. 3.6.5 Quantile diagrams from ‘**limited**’ (⚫) and ‘**unlimited**’ (🞅) simulations for synchronous (left) and parametric (right) resonance cases |

3.6.5.10 This means that simulations that are too long lead to deviation from the Poisson process in relevant sea states, i.e. that the notion of *failure rate* and the formulae from sections 2 and 3 are not applicable. Moreover, using these formulae despite that, i.e. as if the process were a Poisson process, would lead to an under-estimation of the failure rate, i.e. a non-conservative error that should be avoided. Thus, the maximum duration of simulations should be limited. When at least 103 frequencies per wave direction are used, simulations up to 3 hours are acceptable; in general, quantile plots can be used to verify the absence of self-repetition effects.

3.6.5.11 To check whether the described measures always ensure applicability of the Poisson process assumption, the χ2 goodness-of-fit test was applied to several cases of parametric and synchronous resonance in head and beam, respectively, waves at systematically varied significant wave height. Only ‘limited’ simulations of 3-hour duration (or until failure if it happened) were used. Random realisations of the same sea state were repeated until about 103 failures were encountered in each sea state. The observed times to failure were compared with the exponential distribution, which was defined using the maximum likelihood estimate for the failure rate , eq. (3.6.13).

3.6.5.12 The full range t ≥ 0 of time to failure was subdivided into k ≥ 5 intervals of equal probability ΔF = Δ(1 − e−rt) = 1/k; the number of intervals was systematically increased up to a maximum k = N/5. The number Oi of the observed times to failure within each interval i was counted, and the expected number Ei was calculated, according to the assumed exponential distribution, as N / k. After that,

.1 the *test statistic* was calculated as

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|  | (3.6.26) |

.2 the *critical value* of the test statistic at the *significance level* α=0.05 was defined as , the value of the *χ*2 distribution at the cumulative probability 1‑*α*=0.95 with the number of degrees of freedom *f*=*k*−*p*−1, where *p*=1 is the number of parameters of the assumed distribution estimated from the sample data;

.3 the results are shown in Fig. 3.6.6 as the ratio *x*/*c*5% depending on the number of intervals *k*: when *x*/*c*5%<1, the *null hypothesis* that the data follow the assumed distribution cannot be rejected at the significance level 5%.

3.6.5.13 Fig.3.6. 7 shows the ratio x / c5% for k = 200 as a function of the sample mean time to failure. For *synchronous resonance*, the Poisson process model is acceptable (at the 5% significance level) in all studied cases. On the other hand, for *parametric resonance*, the results disagree with the Poisson process assumption: marginally at  and greater and increasingly for  decreasing below 2 hours. Note, however, that the χ2 test is considered as very strict when the amount of data is large;

**3.6.6. Practical implementation of direct counting**

3.6.6.1 The procedure of carrying out numerical simulations or model tests and counting of stability failures should ensure that stability failures satisfy the assumptions of a stationary Poisson process, i.e. it should avoid

.1 self-repetition effects since they lead to a non-conservative assessment: the duration of simulations should be limited by 3 hours if not less than 103 frequencies per wave direction are used for the discretisation of the wave energy spectrum; otherwise, quantile plots can be used to verify the absence of self-repetition effects;

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| Fig. 3.6.6 Ratio of χ2-test statistic to critical value  for significance level α = 5% vs. number of intervals k of time to failure for synchronous (left) and parametric (right) resonance for several values of sample mean time to failure |

.2 transient hydrodynamic effects at the beginning of simulations since they lead to a loss of accuracy: during initial transients, the counter of stability failures and the simulation timer should be switched off; the duration of initial transients can be assumed 50 roll periods; and

.3 auto-correlation of big roll motions. This effect leads to an over-estimation of the sample mean stability failure rate but also under-estimation of the size of the confidence interval. An effective way to avoid the auto-correlation effects is to stop a simulation after the first stability failure.

3.6.6.2 This implementation applies simulations of ship motions in multiple independent realisations of the same irregular seaway. Each simulation can be run for arbitrary time but is limited by the maximum duration and stopped after the first stability failure. During the initial 50 roll periods, the counter of stability failures and simulation timer are switched off. The procedure provides the estimate of the upper boundary rU of the 95%-confidence interval of the rate of stability failures, which is used as the criterion in the probabilistic direct stability assessment in design situations as well as (in combination with statistical extrapolation) in the full probabilistic direct stability assessment and in operational measures.

3.6.6.3 If a stability failure is encountered during a simulation, the total number of failures N is increased by 1 and the total simulation time tt is increased by the time to failure; if a stability failure did not happen during simulation, N is not increased and tt is increased by the simulation time. After that, the sample mean time to failure is calculated as  and the maximum likelihood estimate of the failure rate is calculated as .

3.6.6.4 Using the calculated value , the upper boundary of the 95%-confidence interval of failure rate is calculated as , eq. (17), and compared with the acceptance standard: once  is less than the acceptance standard, further simulations are not required and the loading condition is considered as acceptable in the considered situation.

3.6.6.5 To avoid unnecessary simulations, it is useful to estimate also the lower boundary of the 95%-confidence interval of the failure rate , eq. (3.6.18): once rL exceeds the standard in at least one design situation, the loading condition can be considered as *not acceptable* without further simulations or tests.

3.6.6.6 In such situations when the stability failure rate is low, the required simulation or testing time until the first stability failure may be very large, thus continuing simulations until the first stability failure may become unpractical since the absence of stability failure during a sufficiently long time perhaps means that the loading condition is acceptable. To address such cases, for the first as well as following simulations, after each simulation in which a stability failure *did not happen*, calculate conservative estimates N\*=N+1,  and , and compare the resulting conservative estimate  with the acceptance standard: once  is less than the acceptance standard, the loading condition can be considered as acceptable in the considered situation without further simulations.

**3.6.7 Practical implementation of design assessment in design situations**

3.6.7.1 In the *probabilistic assessment in design situations*, the *acceptance* requirement is that in all design sea states the upper boundary of the 95%-confidence interval of the failure rate should not exceed a *standard* λ (note that since the remaining 5% outside of the confidence interval include both tails, this means that stability failure does not happen with the probability 97.5%). This implementation estimates the upper boundary of the 95%-confidence interval of the failure rate and compares it with the standard as *acceptance check*. To save simulation or model testing time, the procedure also estimates the lower boundary of the 95%-confidence interval of the failure rate and stops further simulations or tests (considering the loading condition as *not acceptable*) once this estimate exceeds the standard in at least one design situation.

3.6.7.2 The procedure uses simulations of ship motions in multiple independent realisations of the same irregular seaway. To ensure that stability failures in numerical simulations or model tests satisfy the assumptions of a Poisson process, the procedure addresses self-repetition effects by limiting the maximum simulation time for each realisation, transient hydrodynamic effects at the beginning of simulations by switching off the counter of stability failures and simulation timer during the initial 50 roll periods and auto-correlation of big roll motions by stopping a simulation after the first stability failure. Apart from limiting the duration and stopping after the first stability failure, each simulation can be run for an arbitrary time.

3.6.7.3 The upper *r*U and lower *r*L boundaries of the 95%-confidence interval of the failure rate can be estimated from eq. (3.6.17) and (3.6.18), respectively, with α=0.05. For *acceptance*, require that rU < λ, which, combined with eq. (3.6.17), provides the following condition:

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|  | (3.6.27) |

3.6.7.4 In eq. (3.6.27), β1 is defined as ; β1 is shown as a function of N in Fig. 3.6.8. The function  can be calculated with MS Excel as chisq.inv.rt(α/2;2\*N).

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| Fig. 3.6.7 Ratio x/c5% for synchronous (⭘) and parametric (▲) resonance vs. sample mean time to failure at k = 200 | Fig. 3.6.8 Functions β1(N) and β2(N) |

3.6.7.5 For *not acceptance*, it can be required that rL < λ, which, combined with eq. (3.6.18), leads to the following condition:

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|  | (3.6.28) |

3.6.7.6 In eq. (3.6.28), , Fig. 8; the function  was calculated with MS Excel as chisq.inv(α/2;2\*N).

3.6.7.7 Note that eq. (3.6.27) is very inconvenient for practical use when the stability failure rate is very low, and thus the required simulation time until the first stability failure is very large. However, continuing simulations until the first stability failure in such cases is senseless since the absence of a failure during a long simulation time is a sign of acceptable safety level. To address such cases, use eq. (3.6.27) with conservative assumptions N = 1 and, correspondingly, , which yields that simulations can be stopped with the *acceptance* decision when the simulation time without failure satisfies the following condition:

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| --- | --- |
|  | (3.6.29) |

3.6.7.8 To extend this idea on the second and further failures, introduce in eq. (3.6.27) the definition eq. (3.6.10), rewritten as , where  is the sample mean time to failure defined from the previous N − 1 stability failures, and assume, conservatively, that TN = t, which leads to the result that that a simulation can be stopped before the N-th failure occurrence with the *acceptance* decision when the simulation time after the N − 1-th failure is

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|  | (3.6.30) |

**3.7 Detailed description of critical wave method for broaching failure method**

3.7.1 The procedures for critical wave method as one of extrapolation procedures of the direct stability assessment to be used for the broaching stability failure is shown here.

3.7.2 The critical wave method is a combination of the probabilistic evaluation of “no-rare” process and the deterministic evaluation of the “rare” process. The “no-rare” process can be regarded as the process of initial condition of the “rare” process. The "non-rare" procedure is evaluation or estimation of probability of encounter of a single large wave that is characterized by exceedance of values of parameters while initial conditions belong to a specified range and the "rare" procedure is the determination of the parameters of single wave and initial conditions that lead to stability failure.

3.7.3 For broaching associated with surf-riding, we may assume that broaching is a single wave event. This is because surf-riding can be regarded as a single wave event. As well established in nonlinear dynamics, surf-riding in regular following waves has two different types: one type occurs under any initial state of surge displacement and velocity if the wave and operational conditions satisfy a critical condition and the other does under the limited initial state of surge displacement and velocity. The latter means that, if a ship is initially placed on a stable surf-riding state for example, a ship keeps the surf-riding for ever. Because of a two-dimensional nature of the phase plane of dynamics, a self-propelled ship cannot enter the initial state for the latter surf-riding without additional forcing other than assumed waves. Therefore, if a ship keeps a specified propeller revolution with the initial propeller revolution is lower than the specified one, the ship cannot experience of the latter type of surf-riding so that the initial condition dependence of surf-riding in regular waves can be excluded. In case of irregular waves, possibility of the former type of surf-riding may exist but is negligibly small because of existing numerical investigations. Further investigations also confirmed that evaluation of broaching probability can be satisfactorily evaluated. Thus, we may ignore the effect of initial conditions. This approach is already adopted in the level 2 vulnerability criteria for surf-riding as shown in 2.6.3 of the Guidelines.

3.7.4 Firstly, the combinations of wavelength and the wave steepness leading to the first-type surf-riding in regular following waves should be determined by using the Melnikov analysis, which is adopted in the level 2 vulnerability criteria as 2.6.3.4.6 of the Guidelines, or its equivalent, under the specified nominal Froude number and the autopilot course from the wave direction.

3.7.5 Secondly, the numerical simulation based on a surge-sway-yaw-roll coupled model with static heave, pitch and an autopilot or equivalent in regular stern quartering waves should be executed for various wavelength to ship length ratio and various wave steepness inside the region of the first-type surf-riding. Here the initial conditions of the ship motions should be set to be a periodic state under a small Froude number such as 0.1 and a small autopilot course from the wave direction such as 0 degrees. The proportional gain for the auto pilot should be set as a practical but reasonably large value, such as 3, the differential gain should be the minimum for avoiding a directionally unstable phenomenon in calm water. The integral gain, the nonlinear elements and the band pass filter of the autopilot may be excluded.

3.7.6 If the instant that both the yaw angle and yaw angular velocity increases over time despite the maximum opposite application of rudder deflection exists, it can be identified as a broaching event. Further, if the roll angle exceeds 40 degrees during this wave encounter, this combination of the wavelength and the wave steepness should be regarded as a stability failure condition due to broaching in regular waves.

3.7.7 Thirdly, the joint probability density function of wavelength and wave height in stationary irregular waves with the significant wave height and the mean wave period is integrated inside the stability failure condition due to broaching in regular waves. The obtained value indicates the conditional probability of the stability failure due to broaching in stationary irregular waves when the ship meets a zero-crossing wave for the specified significant wave height and the mean wave period under the specified nominal Froude number and the autopilot course. The probability density function and the numerical integration method to be used here can be found in paragraphs 3.3.2 and 3.3.4 in annex 3 to of document SDC 6/5.

3.7.8 Repeating the above procedures for various significant wave height and the mean wave period and integrating the product of the above obtained conditional probability and the joint probability density function of the significant wave height and the mean wave period, the year-averaged conditional probability of the stability failure due to broaching when the ship meets a zero-crossing wave under the specified nominal Froude number and the autopilot course. The joint probability density function of the significant wave height and the mean wave period and the numerical integration method to be used here can be found in 2.5.3 of the Guiddelines.

3.7.9 Repeating the above procedures for various autopilot courses and integrating the product of the above obtained conditional probability and the probability density function of the autopilot courses, the year-averaged conditional probability of the stability failure due to broaching when the ship meets a zero-crossing wave under the specified nominal Froude number.

3.7.10 For the specified nominal Froude number, the probability of stability failure occurrence per ship year, *P*, can be calculated as follows:

 (3.7.1)

where *p* and *Twe* are the year-averaged conditional probability of the stability failure due to broaching when the ship meets a zero-crossing wave under the specified nominal Froude number and the mean encounter period under the specified nominal Froude number, respectively.

3.7.11 If the probability of stability failure occurrence per ship year, *P*, is larger than the standard specified in 3.5.3.2.2, the ship operating with the nominal Froude number is judged as unsafe with respect to broaching.

**3.8 Application example of critical wave method for surf-riding/broaching failure method**

**3.8.1 BACKGROUND**

3.8.1.1 This is an application example of critical wave method for surf-riding/broaching failure method based on 3.7.

**3.8.2 SUBJEXT SHIP USED**

3.8.2.1 The subject ship used here is the ONR flare topside vessel as shown in Fig. 3.8.1 Since the ship length is 154 m and the service Froude number is 0.4, this ship shall be regarded vulnerable to broaching in the level 1 vulnerability criteria. The details of this vessel is open for the correspondence group members under the permission of US office of Naval Research and the vessel are known to be vulnerable to broaching danger.

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Fig. 3.8.1 Body plan of the ONR flare topside vessel.

**3.8.3 Used prediction tool**

3.8.3.1 The used prediction tool enables us to calculate broaching probability when a ship meets a wave in the North Atlantic, is a combination of a surge-sway-yaw-roll simulation model with a proportional autopilot in regular waves and a stochastic wave theory. Firstly, the simulation model estimates the deterministic broaching zone for a subject ship as a function of wave steepness and wavelength for the given propeller revolution and the autopilot course from the wave direction. Secondly, the broaching probabilities in stationary sea states specified with the significant wave height, mean wave period and wave spectrum shape are calculated by integrating the probability density function of the local wave height and the wavelength within the deterministic broaching zone. Finally, the annual broaching probability in a specified water area can be obtained by integrating the product of the broaching probabilities in stationary sea states and the occurrence probability of the sea states.

3.8.3.2 The simulation model used here is based on a nonlinear manoeuvring model with the wave-induced forces and moments under the low encounter frequency assumption. The manoeuvring forces and moments, including resistance and propeller thrust, in calm water are estimated with circular motion captive model tests. The roll damping coefficient is estimated with the roll decay tests of the geometrically scaled ship model. The wave-induced forces and moments in stern quartering waves are calculated with a linear slender body theory under the low encounter frequency assumption as the sum of the Froude-Krylov components and hydrodynamic lift due to wave particle velocity, and corrected with captive model tests or computational fluid dynamics of viscos flow. The interactions between the manoeuvring forces and waves are ignored under the assumption that the wave steepness and the ship motion normalised with the forward velocity is not so large. The stochastic wave theory used here was proposed by Longuet-Higgins using a wave envelop theory as used in the level 2 vulnerability criteria.

3.8.3.3 Broaching is defined as a phenomenon that ship cannot keep her straight course regardless the maximum opposite steering efforts. Based on this definition, the following judging criterion is used here:

When the rudder deflection angle reaches its starboard limit, both the ship yaw angular velocity and acceleration have the signs for port;

When the rudder deflection angle reaches its port limit, both the ship yaw angular velocity and acceleration have the signs for starboard.

It is noteworthy here that this criterion does not include any quantitative value for course deviation from the autopilot course.

3.8.3.4 The paper also presents a comparison between the broaching probability calculated with the above method and that measured in model experiments in irregular waves, which is based on the ITTC recommended procedure 7.5-02-07-04 for intact stability model tests. Examples of the comparisons are shown in Figs. 3.8.2-3. These results indicate that the used prediction procedure can be applied to the direct stability assessment of the ONR flare topside vessel.

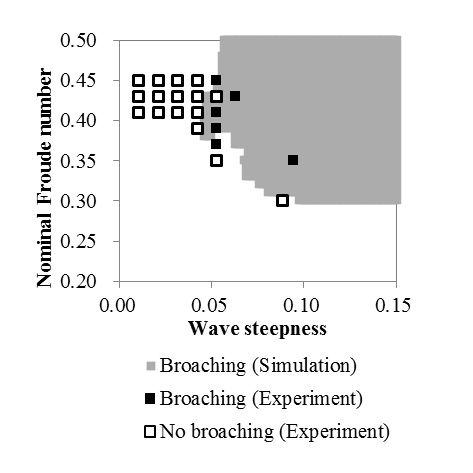


Fig. 3.8.2 Comparison of the experiment and simulation in broaching region for the ONR flare topside vessel in regular waves. Here, the wavelength to ship length ratio is 1.25. The autopilot course is 15 degrees from the wave direction.

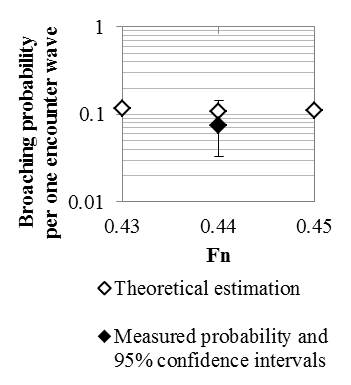


Fig. 3.8.3 Comparison of the experiment and simulation in broaching probability for the ONR flare topside vessel in long-crested irregular waves. Here, the significant wave height and mean wave period are 9.65 m and 11.11 s, respectively. The autopilot course is 15 degrees from the wave direction.

3.8.3.5 In addition, an example of comparison in time series between the model experiment and numerical simulation is shown in Fig. 3.8.4. Here, two broaching instances occurred, approximately 67 s and 97 s into the experiments. The simulation reproduces these two broaching and predicts roll angle due to broaching slightly conservatively.



Fig. 3.8.4 A comparison between the model experiment and the numerical calculation.

**3.8.4 RESULTS AND DISCUSSION**

3.8.4.1 By using the above prediction procedure, the probabilities of roll exceeding 40 degrees due to broaching associated with surf-riding for stationary sea states as the function of the significant wave height, Hs, and the mean wave period, T01 for the different autopilot courses from the wave direction under the nominal Froude number of 0.4 are shown in Tables 1-6. The spectrum shape is based on the ITTC recommendation (1974) and the wave scatter diagram is the IACS No. 34. The Froude number corresponds to her service speed, is 0.4. The rudder gain for the autopilot is 3.

3.8.4.2 The calculations executed here also include the probabilities for different nominal Froude numbers. The effect of the autopilot course is shown in Fig. 3.8.4. It shows that broaching danger exists in the autopilot course ranging from 10 to 30 degrees. If we assume the uniform course distribution, the broaching probability in the North Atlantic is 0.000202. This is the conditional probability of the roll angle exceeding 40 degrees due to broaching associated with surf-riding when the ship meets a zero-crossing wave. As a result, the probability of such failure for one year in the North Atlantic is close to 1. Thus, this assessment concludes that this ship is judged as dangerous for broaching if she is operated without any operational care. Since this ship is judged as vulnerable to broaching failure in the vulnerability level 1 and 2 criteria, this result can be regarded as consistent. The operational guidance can be developed with Tables 3.8.1-6 if we specify the acceptable danger probability for each stationary sea state. Figs. 3.8.4 and 3.8.5 indicates that reducing the nominal forward speed and increasing the threshold of roll angle are effective for decreasing the failure probability of broaching.

Table 3.8.1 Probabilities of roll angle exceeding 40 degrees due to broaching in stationary sea states with the autopilot course of 3.75 degrees with the nominal Froude number of 0.4.



Table 3.8.２ Probabilities of roll angle exceeding 40 degrees due to broaching in stationary sea states with the autopilot course of 7.5 degrees with the nominal Froude number of 0.4.



Table 3.8.3 Probabilities of roll angle exceeding 40 degrees due to broaching in stationary sea states with the autopilot course of 11.25 degrees with the nominal Froude number of 0.4.



Table 3.8.4 Probabilities of roll angle exceeding 40 degrees due to broaching in stationary sea states with the autopilot course of 15 degrees with the nominal Froude number of 0.4.



Table 3.8.5 Probabilities of roll angle exceeding 40 degrees due to broaching in stationary sea states with the autopilot course of 18.75 degrees with the nominal Froude number of 0.4.



Table 3.8.6 Probabilities of roll angle exceeding 40 degrees due to broaching in stationary sea states with the autopilot course of 22.5 degrees with the nominal Froude number of 0.4.



Table 3.8.7 Probabilities of roll angle exceeding 40 degrees due to broaching in stationary sea states with the autopilot course of 26.25 degrees with the nominal Froude number of 0.4.



Table 3.8.8 Probabilities of roll angle exceeding 40 degrees due to broaching in stationary sea states with the autopilot course of 30 degrees with the nominal Froude number of 0.4



Table 3.8.9 Probabilities of roll angle exceeding 40 degrees due to broaching in stationary sea states with the autopilot course of 33.75 degrees with the nominal Froude number of 0.4



Table 3.8.10 Probabilities of roll angle exceeding 40 degrees due to broaching in stationary sea states with the autopilot course of 37.5 degrees with the nominal Froude number of 0.4



Table 3.8.11 Probabilities of roll angle exceeding 40 degrees due to broaching in stationary sea states with the autopilot course of 41.25 degrees with the nominal Froude number of 0.4



Table 3.8.12 Probabilities of roll angle exceeding 40 degrees due to broaching in stationary sea states with the autopilot course of 45 degrees with the nominal Froude number of 0.4



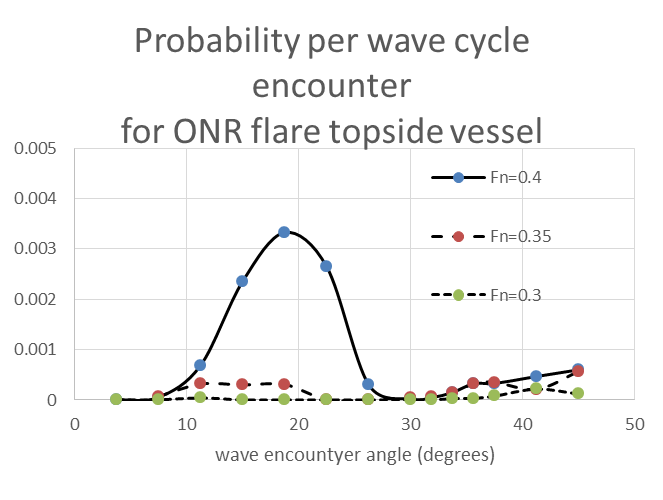


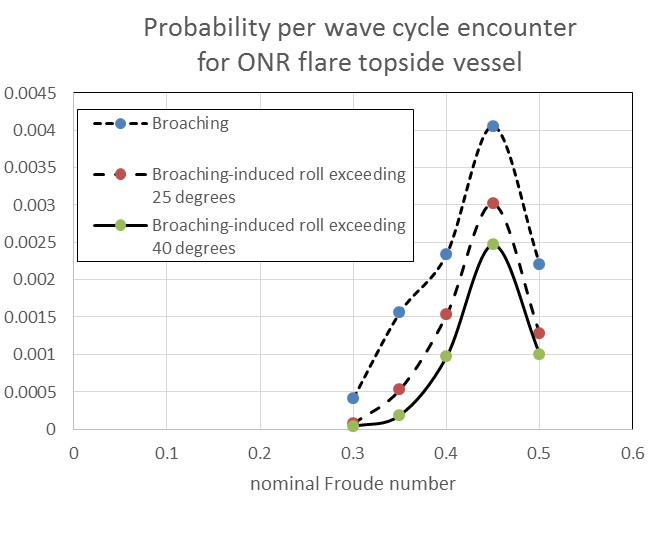
Fig. 3.8.4 Probabilities of roll angle exceeding 40 degrees due to broaching associated with surf-riding as functions of heading angle for the ONR flare topside vessel in the North Atlantic. 

Fig. 3.8.5 Comparison of probabilities relating to broaching associated with surf-riding as functions of the nominal Froude number for the ONR flare topside vessel in the North Atlantic.

**3.9 Validation of critical wave method**

3.9.1 This is a validation study of critical wave method using the direct counting method using an ocean research vessel, which was lost in stern quartering waves off Fukushima in 1980’s.

**3.9.2 Methodology used here**

3.9.2.1 The numerical Code used here is a couple surge-sway-yaw-roll numerical model with quasi-statically estimated heave and pitch as well as a PD autopilot. The calm-water propulsion, roll damping and manoeuvring coefficients were obtained by conventional model tests. The wave forces and moments are estimated as the sum of the Froude-Krylov components and the diffraction components under the low encounter frequency assumption. Empirical correction on the diffraction component in surge was added based on the captive model tests. The autopilot rudder gain was 3 and the time constant for differential control was 0s.

3.9.2.2 The critical wave method can be summarized as follows with numerical examples. Firstly, the surf-riding region in regular following waves is obtained by the Melnikov analysis. An example is shown in Figure 3.9.1. Since the wave celerity for the wavelength to ship length ratio of 1 corresponds to the Froude number of 0.4, surf-riding can occur under this wavelength even with very small wave steepness. Secondly, the broaching-induced large roll zone is determined by repeating time-domain numerical simulations as shown in Figure 3.9.2. Here we can find the critical wave steepness for the stability failure is 0.06 even for the wavelength to ship length ratio of 1. Thirdly, the conditional stability failure probability when a ship meets a zero-cross waves by integrating the joint probability density of local wave steepness and the wavelength, which was based on a wave envelop theory, within the broaching-induced large roll zone.

 Figure 3.9.1 Surf-riding zone for the ocean research vessel with the nominal Froude number of 0.4 in regular waves.

 Figure 3.9.2 The zone of stability failure due to broaching for the ocean research vessel with the nominal Froude number of 0.4 and the autopilot course of 30 degrees from the wave direction in regular waves.

**3.9.3 EXAMPLE OF COMPARISON**

3.9.3.1 Typical example of the stability failure probability estimated by the critical wave method was compared, as shown in Figure 3.9.3, with the direct counting method based on 3.5.4 of the Guidelines and the free-running model experiments based on the ITTC recommended procedures. It indicates that the direct counting method using the numerical code here well agrees with the model experiments. The critical wave method provides the result that is almost identical to the upper limit of the confidence interval of the probability due to the direct counting method. It is reasonable because the critical wave method here does not take account for the initial state effect when the ship meets a critical waves. In other words, we assume that, whenever the ship meets a critical wave, the ship would suffer the stability failure. Thus, conservative estimates can be logically obtained so that the critical wave method is suitable for regulatory purpose. While the direct counting method using a PC requires a few days for one nominal Froude number, the critical wave method does about twenty minutes. In addition, the model experiment using a seakeeping and manoeuvring basin did spend one week. This means that the application of the critical wave method is most feasible for normal design routine.

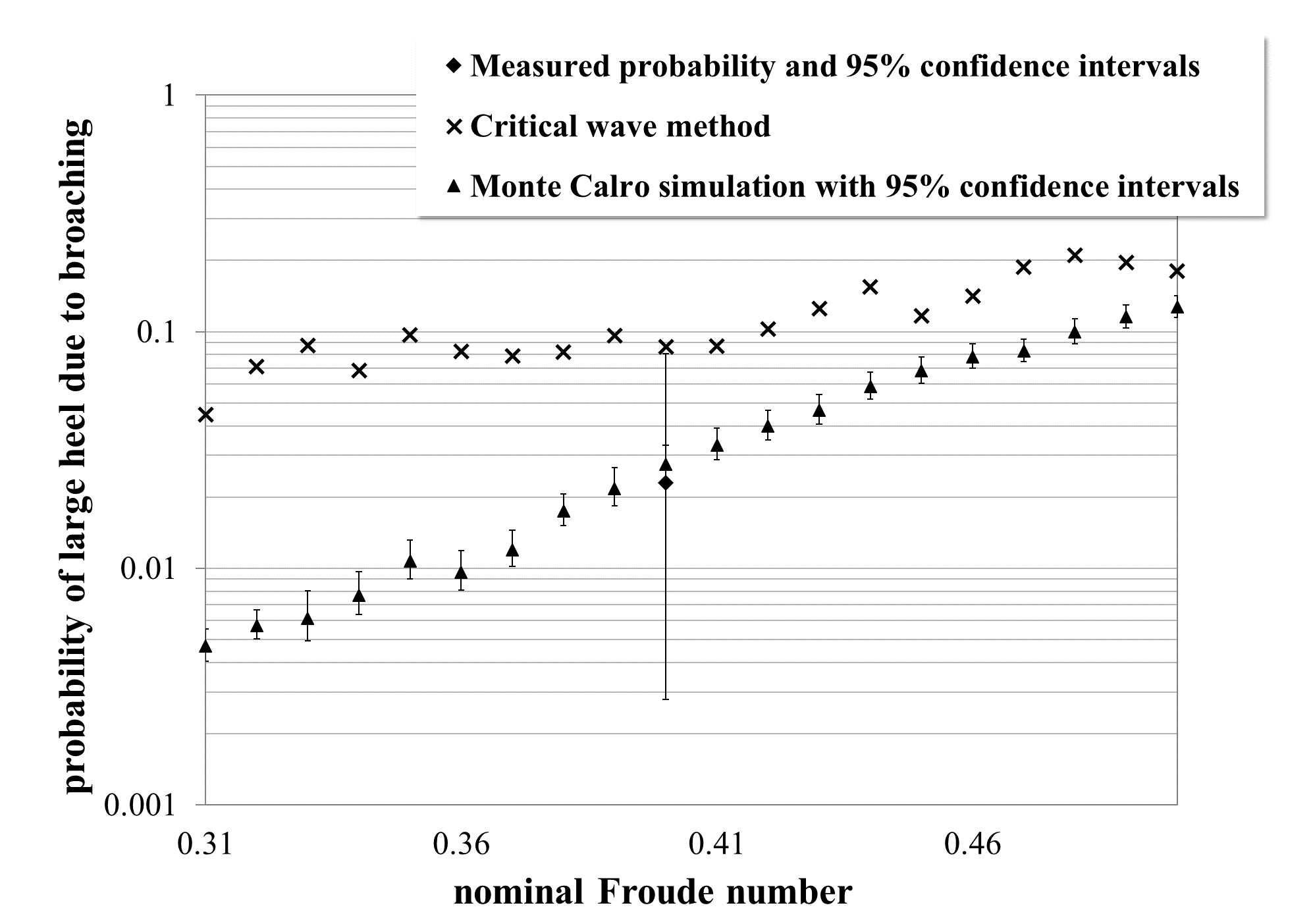


Figure 3.9.3 Comparison in the probability of broaching-induced stability failure for the ocean research vessel with the nominal Froude number of 0.39 and the autopilot course from the wave direction of 30 degrees in irregular waves whose significant wave height is 2.55 m and the mean wave period is 4.52 s.

**3.9.4 CONCLUSIONS**

3.9.4.1 It was demonstrated that direct stability assessment using the critical wave method for stability failure due to broaching is feasible. The preamble/introduction of the interim guidelines should reflect this situation on this particular failure mode.

1. *Shigunov, V. el Moctar, O., and Rathje, H.* (2009) Conditions of parametric rolling, *Proc. 10th Int. Conf. on Stability of Ships and Ocean Vehicles*. [↑](#footnote-ref-1)
2. *Tonguć, E. and Söding, H.* (1986) Computing capsizing frequencies of ships in seaway, *Proc. 3rd Int. Conf. on Stability of Ships and Ocean Vehicles*. [↑](#footnote-ref-2)
3. Derbanne, Q., Storhaug, G., Shigunov, V., Xie, G., and Zheng, G. (2016) Rule formulation of vertical hull girder wave loads based on direct computation, Proc. PRADS 2016, 4th-8th September, Copenhagen, Denmark [↑](#footnote-ref-3)